Stable Skyrmions in Two-Component Bose-Einstein Condensates

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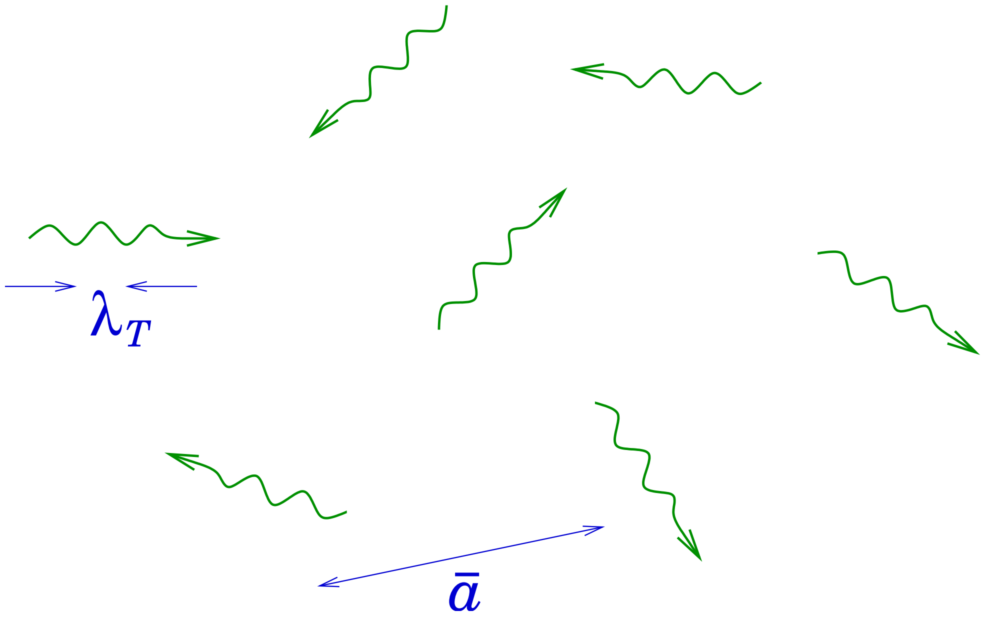
[PRL **88**, 080401 (2002)]

Overview

- Atomic Bose-Einstein Condensates
- Multi-component condensates
- Topological Solitons
- Stable “Skyrmions”
- Summary

Bose-Einstein Condensation



$$\frac{\hbar^2}{m\lambda_T^2} \sim k_B T \qquad \bar{a} \sim \bar{n}^{-1/3}$$

$$\lambda_T \gtrsim \bar{a} \quad \Rightarrow \quad k_B T \lesssim \frac{\hbar^2}{m} \bar{n}^{2/3}$$

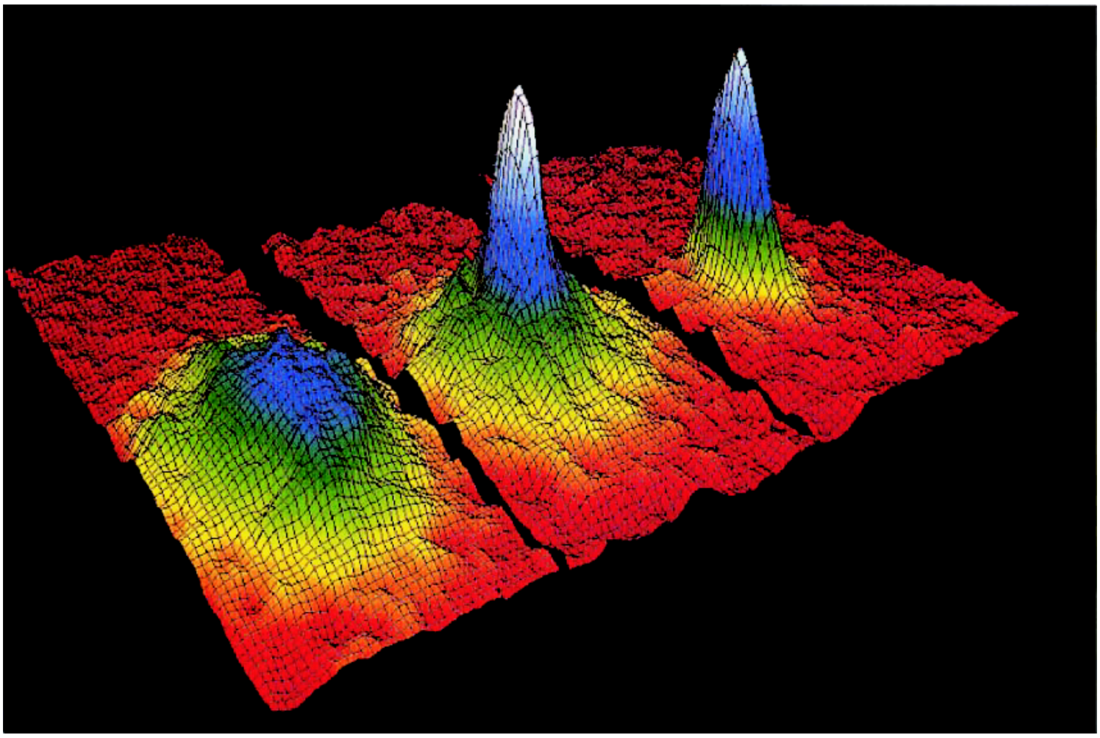
Bosons, $T < T_c \Rightarrow$B.E.C.

$$T = 0 \quad \rightarrow \quad \psi_N = \prod_{i=1}^{N} \psi(\vec{r_i})$$

Atomic Bose-Einstein Condensates

$k_B T_c \simeq \frac{\hbar^2}{m} \bar{n}^{2/3} \sim 100\text{nK}$

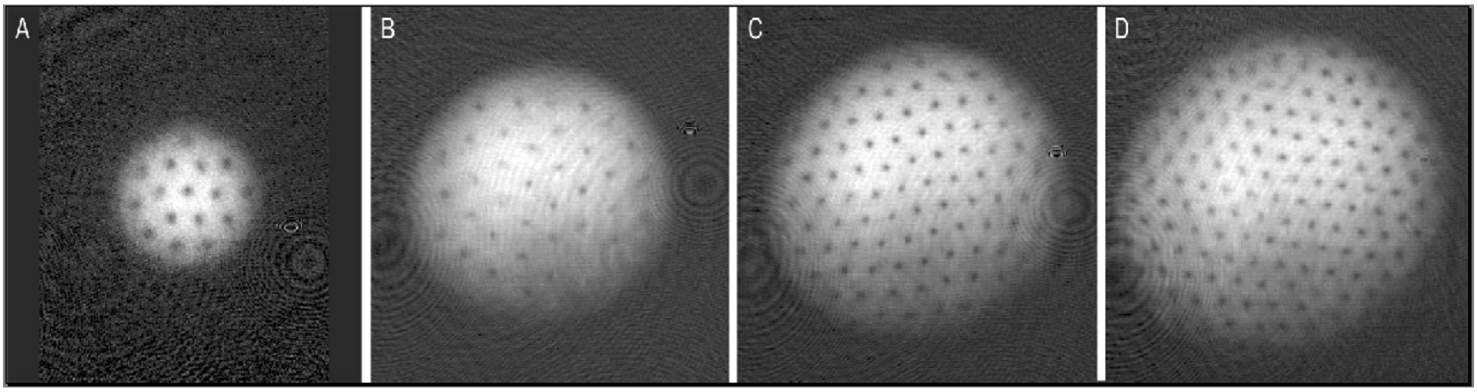
^{87}Rb (JILA, 1995)
^{7}Li (Rice, 1995)
^{23}Na (MIT, 1995)
^{1}H (MIT, 1998)
^{85}Rb (JILA, 2000)
^{4}He* (Orsay, 2001)



[Anderson *et. al.* [JILA], Science **269**, 198 (1995).]

What's special about Atomic BECs?

Fantastic experimental control



[Abo-Shaeer *et al.*[MIT], Science **476**, 476 (2001)]

Tunable effective interactions

s-wave scattering length, a.

atom	a
^{87}Rb	5.77nm
^{7}Li	-1.45nm
^{85}Rb	$-? \to \sim 500$nm

Novel uncondensed states

High vortex density [Wilkin & Gunn,. . .]

Superfluid/(Mott) Insulator transition

[Greiner *et al.*[Munich], Nature **415**, 39 (2002)]

▷ Multi-component systems

Simultaneous trapping and cooling of atoms of different species.

Multicomponent Condensates

Hyperfine interaction $\Rightarrow \vec{F} = \vec{I} + \vec{J}$

Typically: $J = S = 1/2$ (Alkali gases)
$I = 3/2$ (^{87}Rb, ^{23}Na, ^{7}Li)

$$|F = 2, m = -2, -1, 0, 1, 2\rangle$$
$$|F = 1, m = -1, 0, 1\rangle$$

Magnetic trap

^{87}Rb $|F = 2, m = 2\rangle$ and $|F = 1, m = -1\rangle$

[Myatt *et. al.*[JILA], PRL**78**, 586 (1997)]

^{87}Rb $|F = 2, m = 1\rangle$ and $|F = 1, m = -1\rangle$

[Hall *et. al.*[JILA], PRL**81**, 1539 (1998)]

Optical trap

^{23}Na $|F = 1, m = -1, 0, 1\rangle$

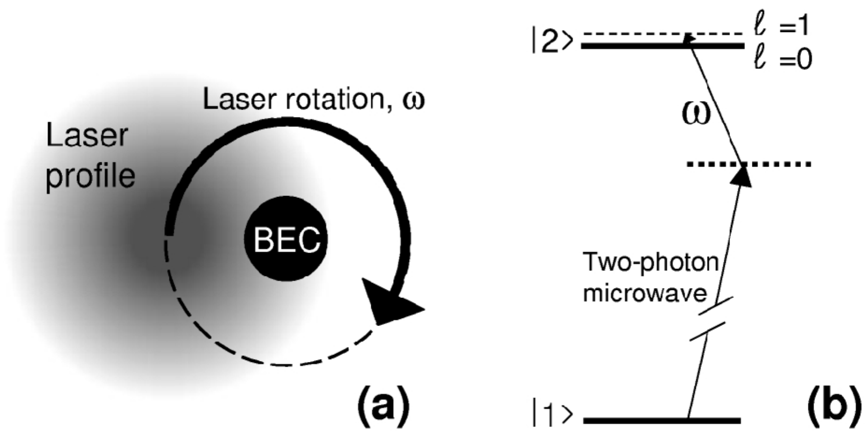
[Stenger *et. al.*[MIT], Nature **396**, 345 (1998)]

Different atoms / Isotopes

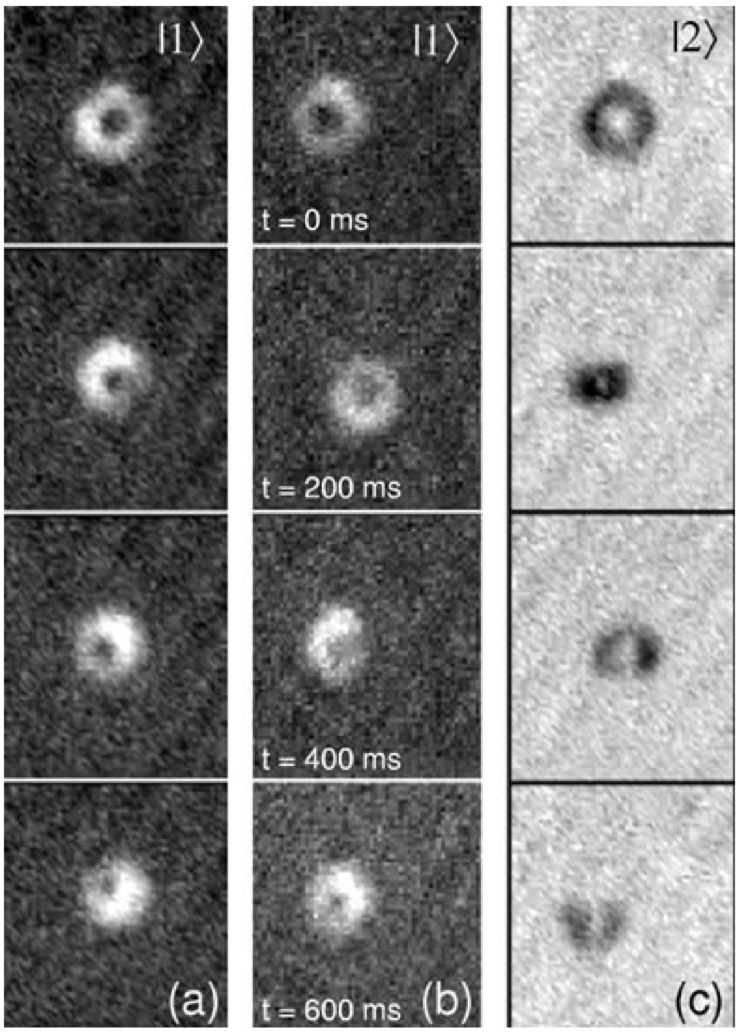
^{85}Rb and ^{87}Rb [Bloch *et. al.*[Munich], PRA **64**, 021402 (2001)]

Vortex in a two-component BEC

[Matthews *et. al.* [JILA], PRL **83**, 2498 (1998).]



$$|1\rangle = |1, -1\rangle$$
$$|2\rangle = |2, 1\rangle$$



Gross-Pitaevskii Mean-Field Theory

$$\Psi_N \propto \prod_{i=1}^{N} \psi(\vec{r}_i)$$
$$N = \int d^3\vec{r}\, |\psi(\vec{r})|^2$$

Minimise the expectation value of the energy

$$E = \int d^3\vec{r} \left[\frac{\hbar^2}{2m} |\nabla \psi|^2 + V(\vec{r}) |\psi|^2 + \frac{1}{2} U |\psi|^4 \right]$$

at fixed N (chemical potential) $\qquad U \propto a$

Multi-component case

$$\Psi_N \propto \prod_{i=1}^{N} \left[\sum_\alpha \psi_\alpha(\vec{r}_i) |\alpha_i\rangle \right]$$
$$N_\alpha = \int d^3\vec{r}\, |\psi_\alpha(\vec{r})|^2$$

Energy (density)

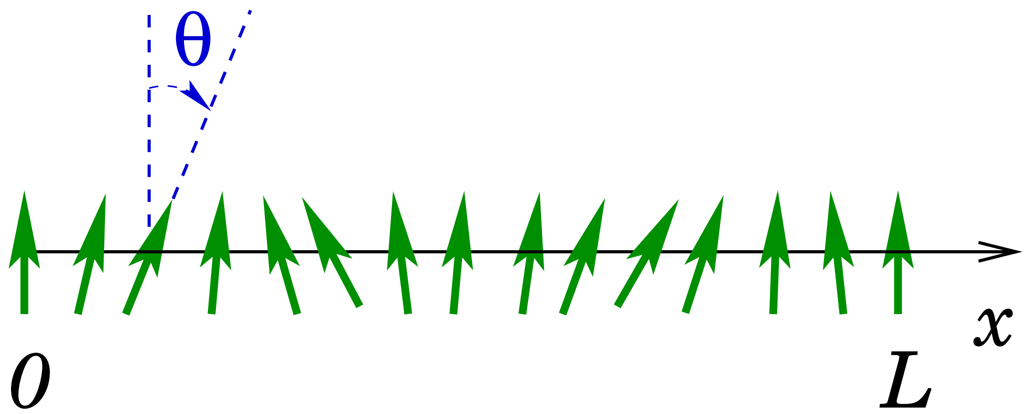
$$\sum_\alpha \frac{\hbar^2}{2m} |\nabla \psi_\alpha|^2 + V_\alpha(\vec{r}) |\psi_\alpha|^2 + \frac{1}{2} \sum_{\alpha,\beta} U_{\alpha\beta} |\psi_\alpha|^2 |\psi_\beta|^2$$

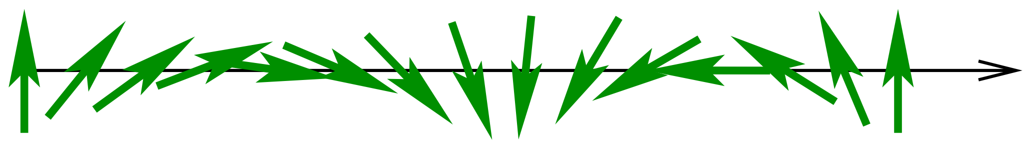
- $U_{\alpha\beta}$ $\Rightarrow$all mutual two-body scattering lengths
- N_α conserved $\Rightarrow$separate chemical potentials
- $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \end{pmatrix}$ $\Rightarrow$topological solitons ?

Topological Textures

One-component condensate in one dimension.

$$\psi(\vec{r}) = \sqrt{\rho}e^{i\theta}$$





"*Topological invariant*"

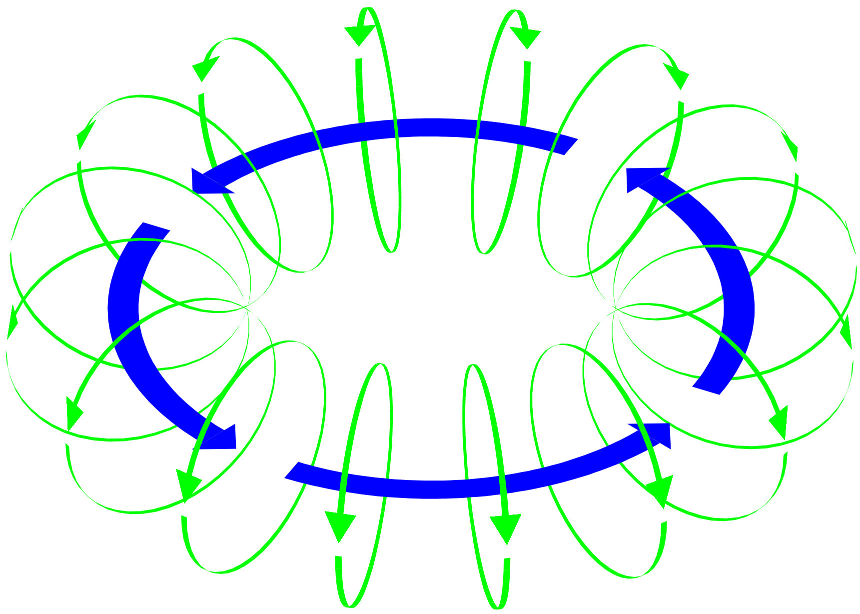
$$Q = \frac{1}{2\pi}\int_0^L \frac{d\theta}{dx}dx$$

Two-component condensate in three dimensions.

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \sqrt{\rho} \begin{pmatrix} \cos(\theta/2)e^{i\phi_1} \\ \sin(\theta/2)e^{i\phi_2} \end{pmatrix}$$

$$Q = \frac{1}{8\pi^2} \int \sin\theta \nabla_i \theta \nabla_j \phi_1 \nabla_k \phi_2 \, \epsilon_{ijk} d^3\vec{r}$$

"Skyrmion"



[Al Kawaja and Stoof, Nature **411**, 918 (2001);
Ruostekoski and Anglin, PRL **86**, 3934 (2001)]

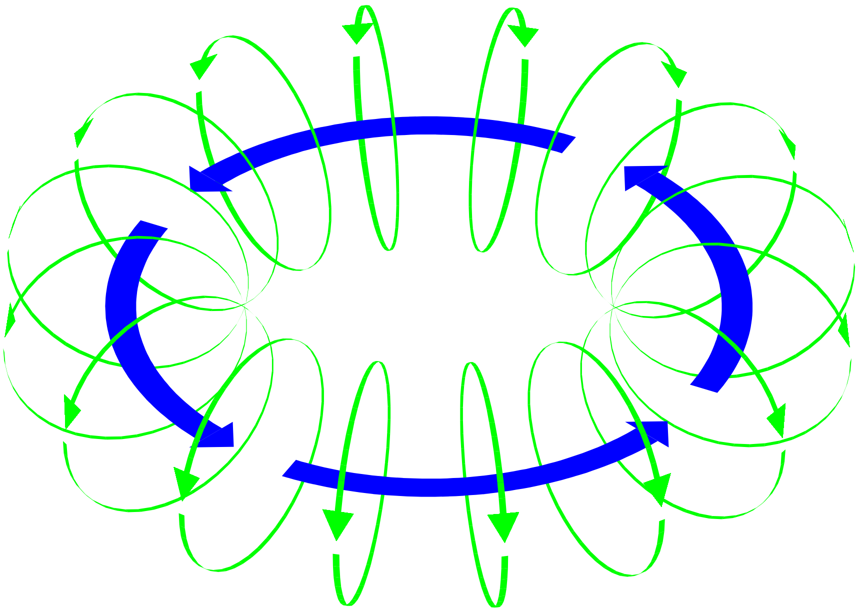
[Three components, textures vortices, monopoles...]

How to obtain stable Skyrmions

- Large Trap $\psi(|\vec{r}| \to \infty) = \sqrt{\rho_0} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
- Constrain $N_2 = \int d^3\vec{r}\, |\psi_2|^2$
- Regime of phase separation: $U_{12}^2 > U_{11}U_{22}$

We study $U_{11} \sim U_{12} \sim U_{22}$ s.t. $\rho(\vec{r}) = \rho_0$.

Find stable skyrmions of the form:



"Imprint" with lasers [Ruostekoski and Anglin, PRL **86**, 3934 (2001)]

[cf. "Cosmic vortons"]

Mathematical Details

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \sqrt{\rho_0} \begin{pmatrix} \cos(\theta/2) e^{i\phi_1} \\ \sin(\theta/2) e^{i\phi_2} \end{pmatrix}$$

$$\begin{aligned} N_2 &= \rho_0 \int \sin^2(\theta/2) d^3\vec{r} \\ Q &= \frac{1}{8\pi^2} \int \sin\theta \nabla_i \theta \nabla_j \phi_1 \nabla_k \phi_2 \, \epsilon_{ijk} d^3\vec{r} \end{aligned}$$

Energy density

$$\frac{\hbar^2 \rho_0}{2m} \left[\frac{1}{4} |\nabla\theta|^2 + \cos^2(\theta/2) |\nabla\phi_1|^2 + \sin^2(\theta/2) |\nabla\phi_2|^2 \right] + \Delta \sin^2\theta$$

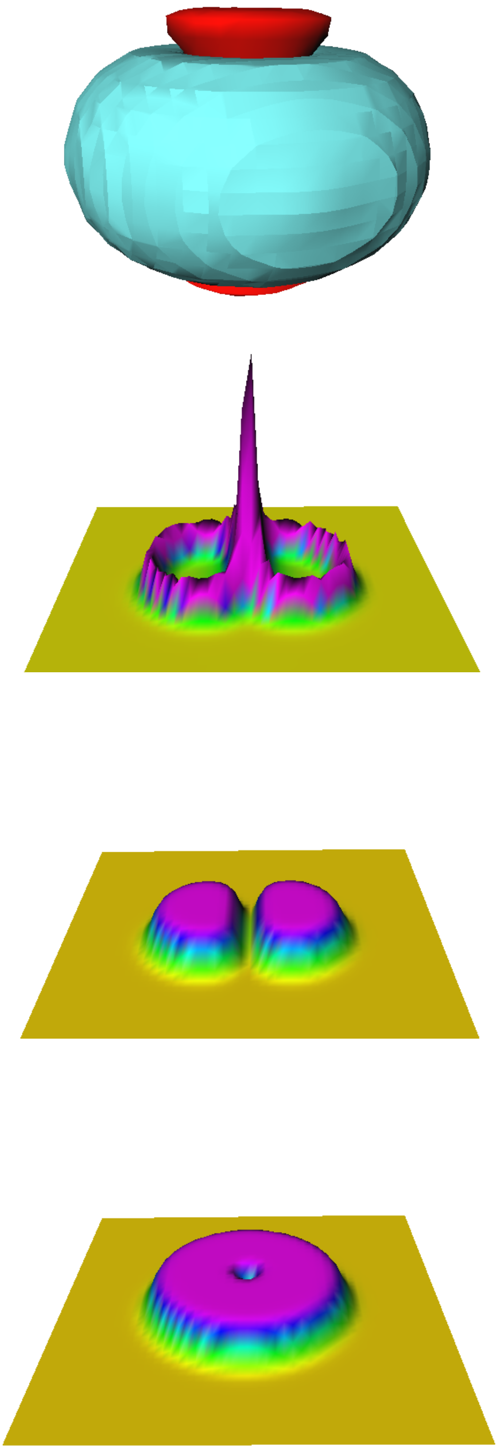
$$\left[\Delta \equiv \tfrac{1}{8} \rho_0^2 \left(2U_{12} - U_{11} - U_{22}\right)\right]$$

Lengthscales: $\xi_\Delta \equiv \sqrt{\frac{\hbar^2 \rho}{2m\Delta}} \quad R_2 = \left(\frac{N_2}{\rho_0}\right)^{1/3}$

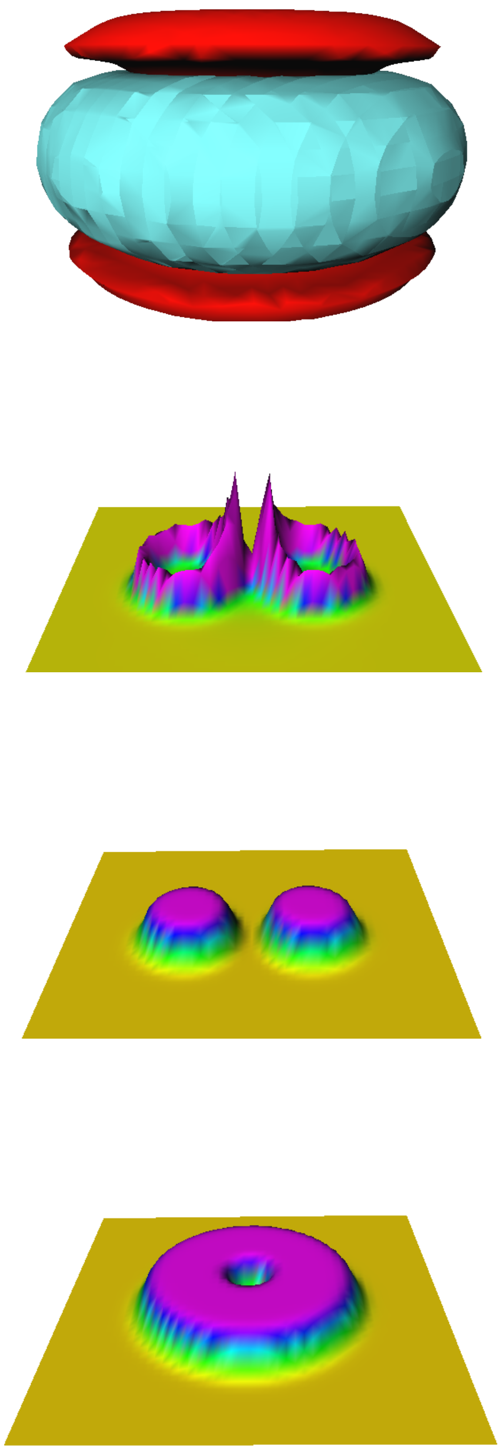
$$E(\Delta, N_2) = \left(\frac{\hbar^2 \rho_0 R_2}{m}\right) \mathcal{E}_Q(\eta)$$

$$\overbrace{\eta \equiv \frac{R_2}{\xi_\Delta}}$$

Q=1

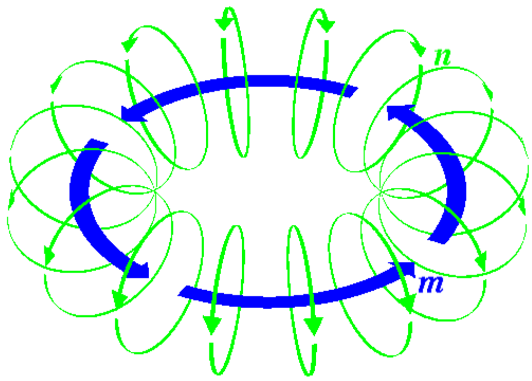


Q=2

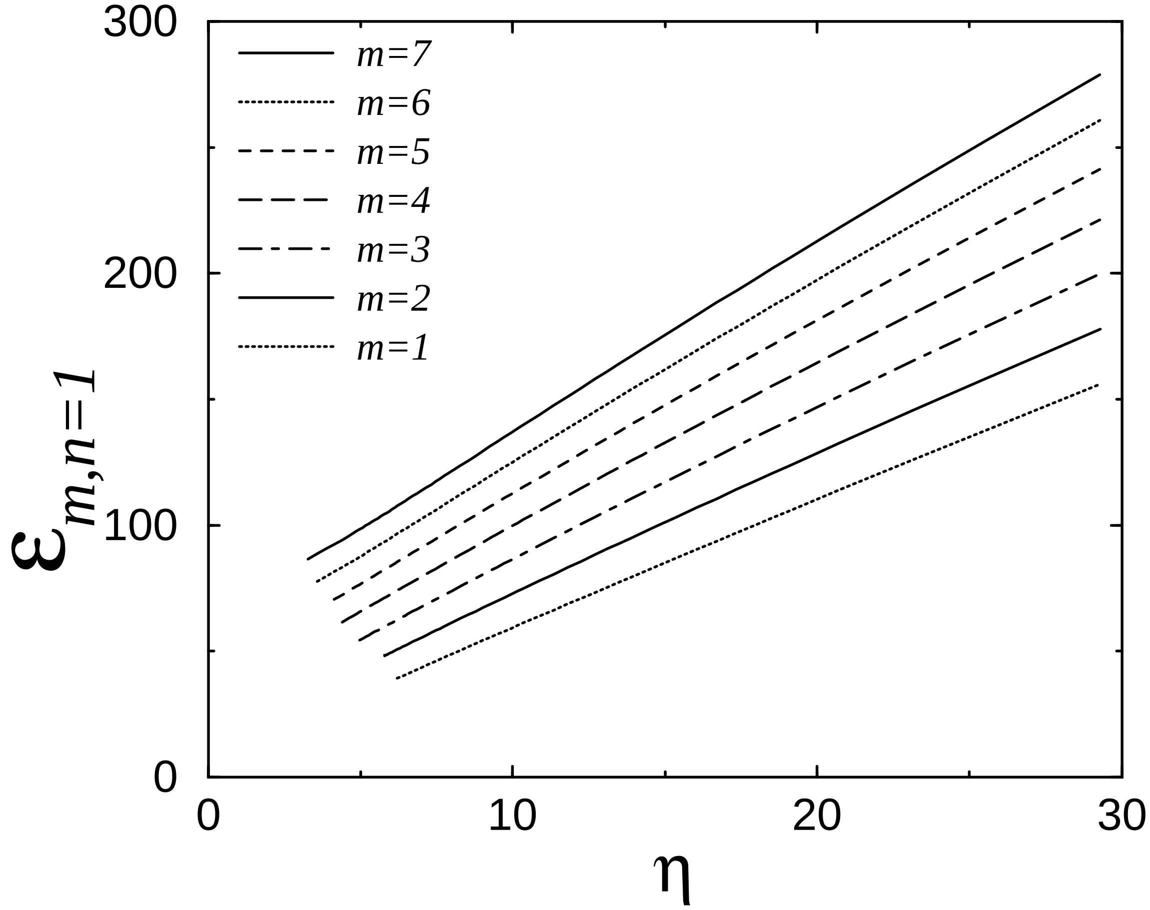


Axisymmetric Ansatz

$\theta(r,z), \phi_1(r,z), \phi_2 = m\chi$
[(r,χ,z) are cylindrical polar co-ordinates]



$Q = mn$

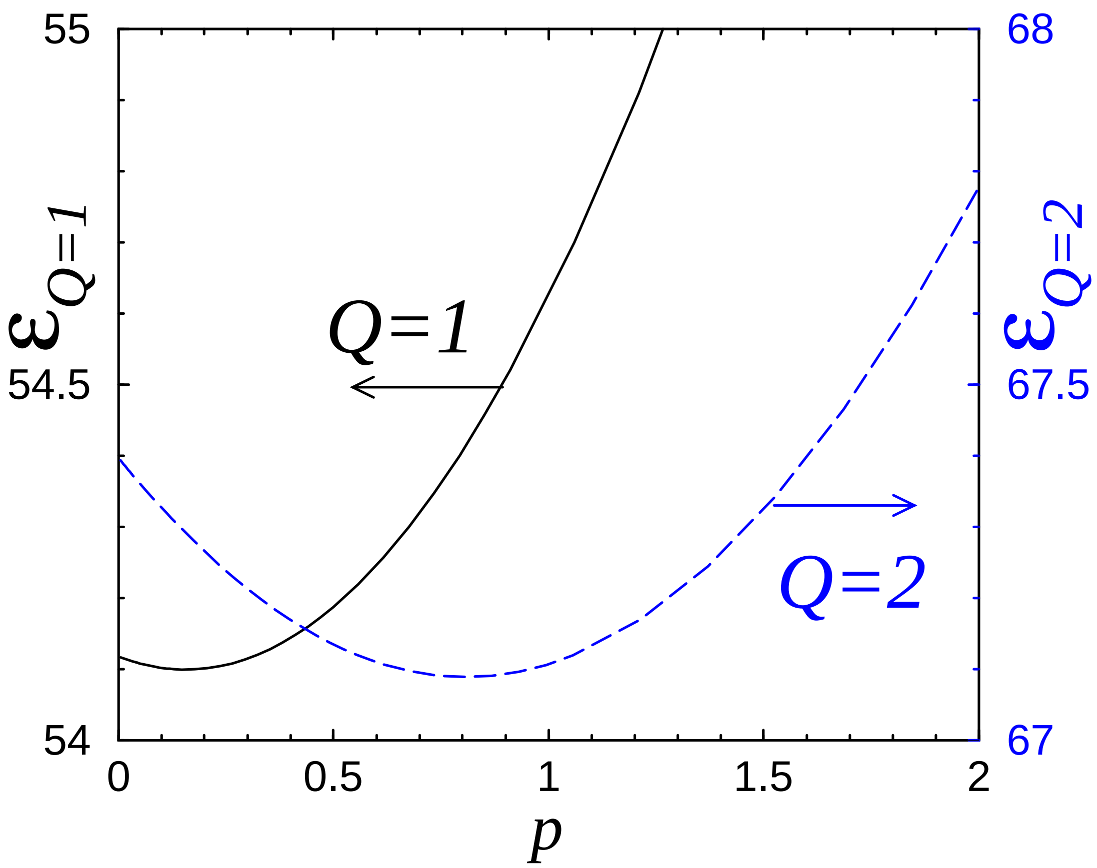


Moving Vortex Rings

Constrain the impulse (momentum)

$$P_i = \frac{\hbar}{2i} \int d^3\vec{r} [r_j \nabla_i \psi_\alpha^* \nabla_j \psi_\alpha - r_j \nabla_j \psi_\alpha^* \nabla_i \psi_\alpha]$$

$$\vec{v} = \frac{\partial E}{\partial \vec{P}}$$



Summary

• Atomic Bose-Einstein condensates offer the possibility of studying interacting Bose gases with a high level of control (interaction strength, confinement, numbers of components).

• In the regime of *phase separation*, two-component BECs have stable textures with the topology of Skyrmions ($Q = 1, 2$).

• This regime is relevant for 2-component ^{87}Rb systems. We expect that textures imprinted by lasers will relax to these stable Skyrmion configurations.