Optical Flux Lattices for Ultracold Atomic Gases

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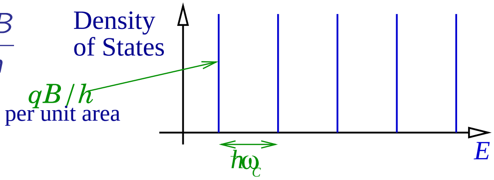
Non-Standard Superfluids and Insulators
ICTP Trieste, 19 July 2011

NRC, PRL **106**, 175301 (2011);
NRC & Jean Dalibard, arXiv:1106.0820;
Benjamin Béri & NRC, arXiv:1101.5610

Motivation: fractional quantum Hall regime

2D charged particle in magnetic field ⇒Landau levels

Highly degenerate $n_\phi = \dfrac{qB}{h}$



FQH states for $n_{2\mathrm{D}} \sim n_\phi$

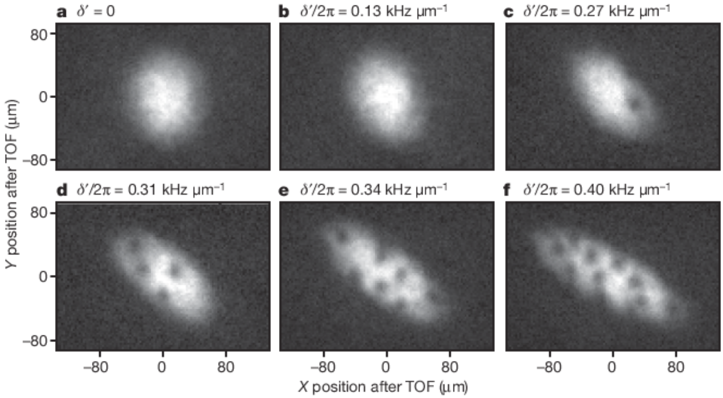
Rotating BECs $n_\phi = \dfrac{2M\Omega}{h}$ Vortex lattice "melts" for $\frac{n_{2\mathrm{D}}}{n_\phi} \lesssim 6$

[NRC, Wilkin & Gunn, PRL (2001)]

But $\Omega \simeq 2\pi \times 100\mathrm{Hz} \Rightarrow n_\phi \lesssim 2 \times 10^7 \mathrm{cm}^{-2}$

Optically Induced Gauge Fields

[Y.-J. Lin, R.L. Compton, K. Jiménez-García, J.V. Porto and I.B. Spielman, Nature **462**, 628 (2009)]



"Optical Flux Lattices"

[NRC, PRL **106**, 175301 (2011); NRC & Jean Dalibard, arXiv:1106.0820]

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \hat{V}(\mathbf{r})$$

- Narrow bands with non-zero Chern number.
  $n_\phi \simeq 10^9 \text{cm}^{-2} \Rightarrow$FQH states at high particle densities.

- Distinct from previous tight-binding proposals.
  [Jaksch & Zoller (2003); Mueller (2004); Sørensen, Demler & Lukin (2005); Gerbier & Dalibard (2010)]

- Generalizes to $\mathbb{Z}_2$ topological invariant. [Benjamin Béri & NRC, arXiv:1101.5610]

- "Nearly free electron" approach to topological insulators.

Outline

Optically Induced Gauge Fields

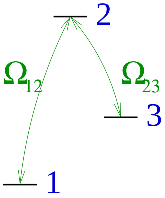
"Optical Flux Lattices"

- One-Photon Implementation
- Two-Photon Implementation

$\mathbb{Z}_2$ Topological Insulators

Optically Induced Gauge Fields

[J. Dalibard, F. Gerbier, G. Juzeliūnas, P. Öhberg, arXiv:1008.5378]



Optical coupling of several internal states

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \hat{V}(\mathbf{r})$$

$\hat{V}(\mathbf{r})$ has local dressed states $|n_\mathbf{r}\rangle$, spectrum $E_n(\mathbf{r})$

$$|\psi(\mathbf{r})\rangle = \sum_n \psi_n(\mathbf{r})|n_\mathbf{r}\rangle$$

Adiabatic motion $H_n\psi_n = \langle n_\mathbf{r}|\hat{H}\psi_n|n_\mathbf{r}\rangle$

$$\hat{H}_n = \frac{(\mathbf{p} - q\mathbf{A})^2}{2M} + V_n(\mathbf{r}) \qquad q\mathbf{A} = i\hbar\langle n_\mathbf{r}|\boldsymbol{\nabla} n_\mathbf{r}\rangle$$

Maximum flux density: Back of the envelope

Vector potential $q\mathbf{A} = i\hbar\langle 0_\mathbf{r}|\boldsymbol{\nabla} 0_\mathbf{r}\rangle \Rightarrow |q\mathbf{A}| \lesssim \frac{h}{\lambda}$

Cloud of radius $R \gg \lambda$

$$\bar{n}_\phi \pi R^2 \equiv \int n_\phi d^2\mathbf{r} = \frac{q}{h}\int \boldsymbol{\nabla}\times\mathbf{A}\cdot d\mathbf{S} = \frac{q}{h}\oint \mathbf{A}\cdot d\mathbf{r} \lesssim \frac{1}{\lambda}(2\pi R)$$

$$\Rightarrow \bar{n}_\phi \lesssim \frac{1}{R\lambda} \simeq 2\times 10^7 \text{cm}^{-2}$$

$[R \simeq 10\mu\text{m} \;\; \lambda \simeq 0.5\mu\text{m}]$

Maximum flux density: Carefully this time!

~~Optical wavelength $\lambda \Rightarrow |q\mathbf{A}| \lesssim \frac{h}{\lambda}$~~

$\mathbf{A}$ can have *singularities*

$$|0_{\mathbf{r}}\rangle = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)e^{-i\phi} \end{pmatrix} \quad \Rightarrow \quad q\mathbf{A} = -\hbar \sin^2(\theta/2)\nabla\phi$$

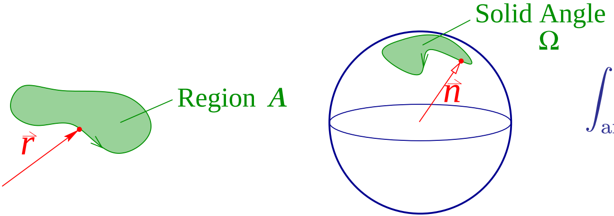
Singularities can appear for $\theta = \pi$.

Vanishing net flux. Can be removed by a gauge transformation.
[e.g. "Dirac strings"]

Gauge-independent approach (two-level system)

Bloch vector $\vec{n}(\mathbf{r}) = \langle 0_\mathbf{r} | \hat{\vec{\sigma}} | 0_\mathbf{r} \rangle$

$$n_\phi = \frac{1}{8\pi}\epsilon_{ijk}\epsilon_{\mu\nu} n_i \partial_\mu n_j \partial_\nu n_k \qquad |n_\phi| \lesssim \frac{1}{\lambda^2}$$



$$\int_{\text{area } A} n_\phi d^2 r = \frac{\Omega}{4\pi}$$

The number of flux quanta in region A is the number of times the Bloch vector wraps over the sphere.

"Optical flux lattices"

[NRC, Phys. Rev. Lett. **106**, 175301 (2011)]

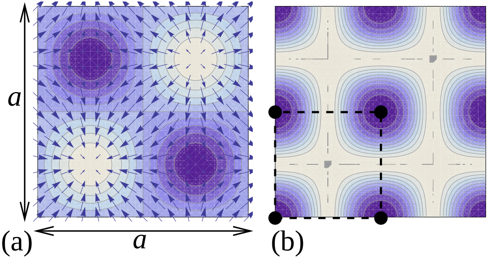
Spatially periodic configurations for which the Bloch vector wraps the sphere a nonzero integer number, N_ϕ, times in each unit cell.

$$\bar{n}_\phi = \frac{N_\phi}{A_{\rm cell}} \sim \frac{1}{\lambda^2} \simeq 10^9 \text{cm}^{-2}$$

vectors (n_x, n_y)

contours n_z

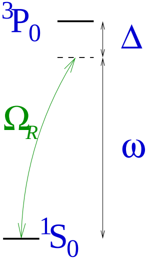
$N_\phi = 2$



contours n_ϕ

Optical Flux Lattice: One-Photon Implementation

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \mathcal{V}\hat{M}(\mathbf{r}) \qquad \hat{M} = \vec{M}(\mathbf{r}) \cdot \hat{\vec{\sigma}}$$



e.g. $^1S_0$ and $^3P_0$ for Yb or alkaline earth atom

[F. Gerbier & J. Dalibard, NJP **12**, 033007 (2010)]

M_x, M_y: Rabi coupling, $\omega \simeq \omega_0$

M_z: standing waves at "anti-magic" frequency, $\omega_{\rm am}$

$$\mathcal{V}\hat{M} = \begin{pmatrix} -\frac{\hbar\Delta}{2} - V_{\rm am}(\mathbf{r}) & \frac{\hbar\Omega(\mathbf{r})}{2} \\ \frac{\hbar\Omega^*(\mathbf{r})}{2} & \frac{\hbar\Delta}{2} + V_{\rm am}(\mathbf{r}) \end{pmatrix}$$

Square Lattice

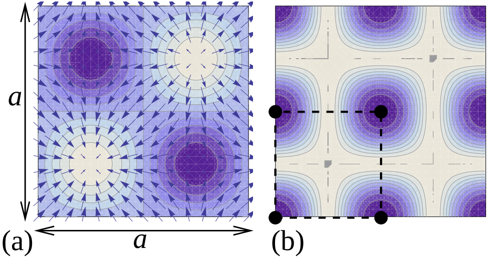
$$\hat{M}_{\rm sq} = \cos(\kappa x)\hat{\sigma}_x + \cos(\kappa y)\hat{\sigma}_y + \sin(\kappa x)\sin(\kappa y)\hat{\sigma}_z$$

where $\kappa \equiv 2\pi/a$.

vectors (n_x, n_y)

contours n_z

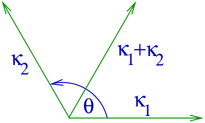
$N_\phi = 2$



contours n_ϕ

Triangular lattice

$$\hat{M}_{\rm tri} = \cos(\mathbf{r}\cdot\boldsymbol{\kappa}_1)\hat{\sigma}_x + \cos(\mathbf{r}\cdot\boldsymbol{\kappa}_2)\hat{\sigma}_y + \cos[\mathbf{r}\cdot(\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)]\hat{\sigma}_z$$

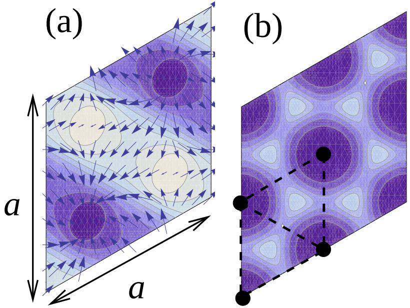


$\theta \simeq 2\pi/3$

vectors: (n_x, n_y)

contours: n_z

$N_\phi = 2$

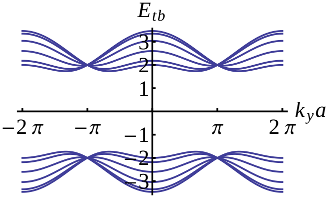


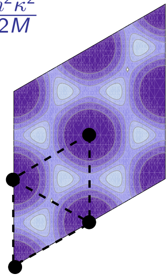
Bandstructure (Triangular Lattice)

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \mathcal{V}\left[c_1\hat{\sigma}_x + c_2\hat{\sigma}_y + c_{12}\hat{\sigma}_z\right]$$

$$c_i \equiv \cos(\boldsymbol{\kappa}_i \cdot \mathbf{r}), c_{12} \equiv \cos[(\boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2) \cdot \mathbf{r}]$$

Tight-binding limit $\quad \mathcal{V} \gtrsim E_R \equiv \frac{\hbar^2\kappa^2}{2M}$

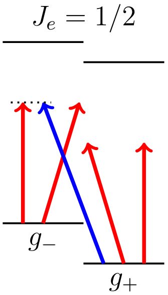




Lowest energy band has narrow width and Chern number of 1.

Two-Photon Dressed States ($J_g = 1/2$)

[NRC & Jean Dalibard, arXiv:1106.0820]



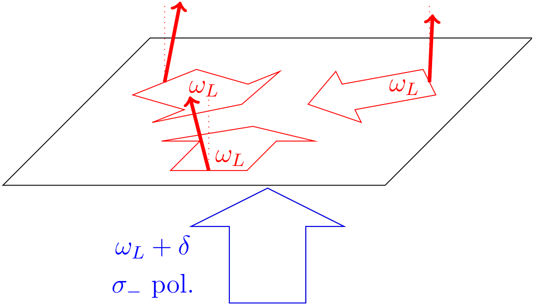
e.g. ^{171}Yb or ^{179}Hg; $g = {}^1S_0$, $e = {}^3P_0$

Light at two frequencies:

- ω_L with Rabi freqs. κ_m ($m = 0, \pm 1$)
- $\omega_L + \delta$ with Rabi freq. E in σ_-

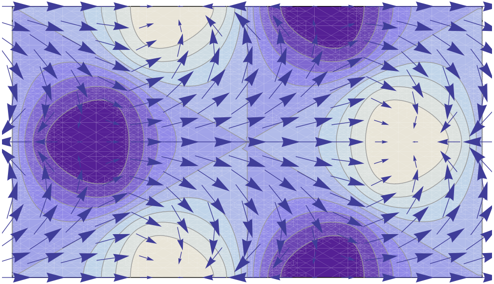
$$\hat{V} = \frac{\hbar\kappa_{\rm tot}^2}{3\Delta}\hat{1} + \frac{\hbar}{3\Delta}\begin{pmatrix} |\kappa_-|^2 - |\kappa_+|^2 & E\kappa_0 \\ E\kappa_0^* & |\kappa_+|^2 - |\kappa_-|^2 \end{pmatrix}$$

Triangular symmetry $$\boldsymbol{\kappa} = \kappa \sum_{i=1}^{3} e^{i\mathbf{k}_i\cdot\mathbf{r}} \left[\cos\theta\,\hat{\mathbf{z}} + \sin\theta\,(\hat{\mathbf{z}}\times\hat{\mathbf{k}}_i)\right]$$



$$\mathcal{V} = \frac{\hbar\kappa^2}{3\Delta}; \quad \theta; \quad \epsilon = \frac{E}{\kappa}$$

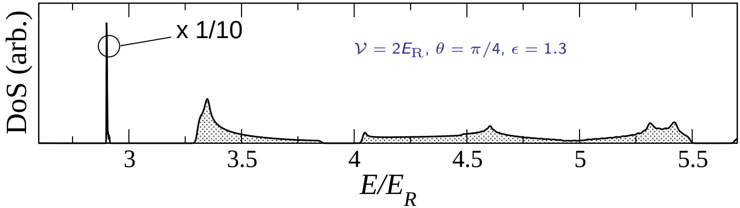
[Under a gauge transformation, $\hat{U} \equiv \exp(-i\mathbf{k}_3 \cdot \mathbf{r}\,\hat{\sigma}_z/2)$...]



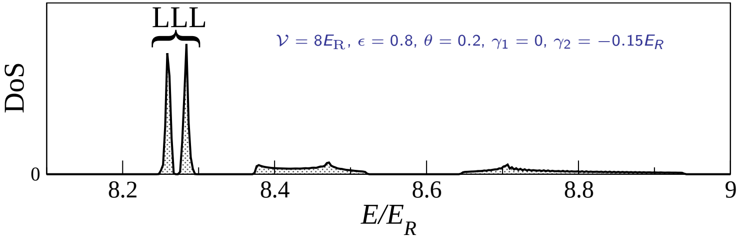
Bloch vector wraps the sphere once in the unit cell.

Two-level system $\Rightarrow N_\phi = 1$ per unit cell.

$J_g = 1/2$ (e.g. ^{171}Yb, ^{179}Hg)



$J_g = 1$ (e.g. ^{23}Na, ^{39}K, ^{87}Rb)



- Narrow topological bands: OFL analogue of lowest Landau level.

Experimental Consequences

Non-interacting fermions (IQHE)

- Filled band has chiral edge state:

Precession of collective modes.

Interacting fermions/bosons

Strongly correlated phases if interactions large compared to bandwidth: likely candidates for FQHE states.

- Incompressible states (density plateaus).
- Chiral edge modes.

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$\mathbb{Z}_2$ Topological Insulators

Topological Insulators

[Hasan & Kane, RMP **82**, 3045 (2010); Qi & Zhang, arXiv:1008.2026]

TI: Band insulator with gapless surface states.

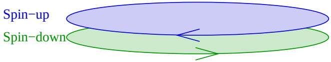
- IQHE: 2D, broken time reversal symmetry (TRS)

  Chern number $\Rightarrow$number of chiral edge states

- $\mathbb{Z}_2$ TI: fermions ($S = \frac{1}{2}, \frac{3}{2}, \ldots$) *with* TRS (Kramers' deg.)

  Band insulators are: trivial; or non-trivial (metallic surface)

  2D: counterpropagating edge channels of opposite spin;



  3D: relativistic (Dirac) 2D surface state.

$\mathbb{Z}_2$ Topological Insulators

$$\hat{H} = \frac{\mathbf{p}^2}{2m}\hat{\mathbb{I}}_N + \mathcal{V}\hat{M}(\mathbf{r})$$

[Benjamin Béri & NRC, arXiv:1101.5610]

Time-reversal symmetry ⇒minimum (interesting) $N = 4$

$$\begin{aligned} \hat{M} &= \begin{pmatrix} (A+B)\hat{\mathbb{I}}_2 & C\hat{\mathbb{I}}_2 - i\hat{\vec{\sigma}}\cdot\vec{D} \\ C\hat{\mathbb{I}}_2 + i\hat{\vec{\sigma}}\cdot\vec{D} & (A-B)\hat{\mathbb{I}}_2 \end{pmatrix} \\ &= A\hat{\mathbb{I}}_4 + B\hat{\Sigma}_3 + C\hat{\Sigma}_1 + \vec{D}\hat{\Sigma}_2\hat{\vec{\sigma}} \end{aligned}$$

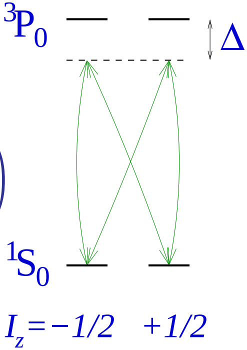
$[A, B, C, \vec{D} = (D_x, D_y, D_z)$ real$]$

Dressed states are Kramers' doublets ⇒non-abelian gauge field.

[Osterloh *et al.* PRL (2005); Ruseckas *et al.* PRL (2005)]

^{171}Yb has nuclear spin $I = 1/2$

$$\mathcal{V}\hat{M} = \begin{pmatrix} -\left(\frac{\hbar}{2}\Delta + V_{\rm am}\right)\hat{\mathbb{I}}_2 & -i\hat{\vec{\sigma}}\cdot\vec{\mathcal{E}}d_r \\ i\hat{\vec{\sigma}}\cdot\vec{\mathcal{E}}^* d_r & \left(\frac{\hbar}{2}\Delta + V_{\rm am}\right)\hat{\mathbb{I}}_2 \end{pmatrix}$$

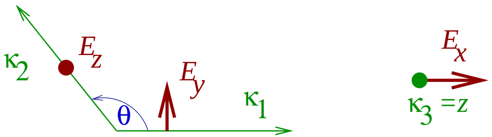


TRS preserved if all components of $\vec{\mathcal{E}}$ have a common phase.

Two Dimensions

$$d_r\vec{\mathcal{E}} = \mathcal{V}\left(\delta, \cos(\mathbf{r}\cdot\boldsymbol{\kappa}_1), \cos(\mathbf{r}\cdot\boldsymbol{\kappa}_2)\right)$$

$$\boldsymbol{\kappa}_1 = (1,0,0)\kappa$$
$$\boldsymbol{\kappa}_2 = (\cos\theta, \sin\theta, 0)\kappa$$



For Yb, $\theta \simeq 2\pi/3$

$$\frac{\hbar}{2}\Delta + V_{\rm am}(\mathbf{r}) = -\mathcal{V}\cos[\mathbf{r}\cdot(\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)]$$

$\hat{U} = 2^{-1/2}(\hat{\mathbb{I}}_4 - i\hat{\Sigma}_3\hat{\sigma}_2)$

$$\hat{M}' = \hat{U}^\dagger \hat{M} \hat{U} = c_1\hat{\Sigma}_1 + c_2\hat{\Sigma}_2\hat{\sigma}_3 + c_{12}\hat{\Sigma}_3 + \delta\hat{\Sigma}_2\hat{\sigma}_1.$$

$$c_i \equiv \cos(\boldsymbol{\kappa}_i \cdot \mathbf{r}), c_{12} \equiv \cos[(\boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2) \cdot \mathbf{r}]$$

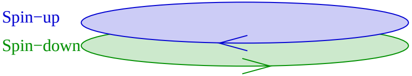
(i) Decoupled spins, $\delta = 0$

$$\hat{M}' = c_1\hat{\Sigma}_1 \pm c_2\hat{\Sigma}_2 + c_{12}\hat{\Sigma}_3$$

OFLs of opposite flux for spin $\sigma_3 = \pm 1$.

$\sigma_3 = \pm 1$ bands are degenerate, but with opposite Chern numbers.

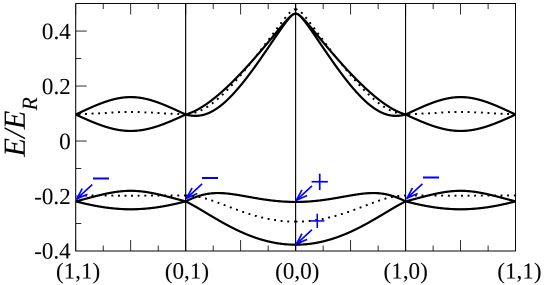
$\Rightarrow$ "quantum spin Hall" system

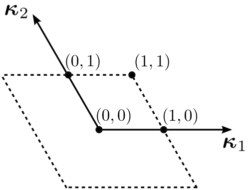


(ii) "Spin-orbit coupling", $\delta \neq 0$

Inversion symmetry [Fu & Kane, PRB (2007)]

$$\Gamma_{nm} = \frac{1}{2}(n\boldsymbol{\kappa}_1 + m\boldsymbol{\kappa}_2) \qquad \prod_{n,m=0,1} \prod_{\alpha \in \text{filled}} \xi_{nm}^{(\alpha)} = -1$$

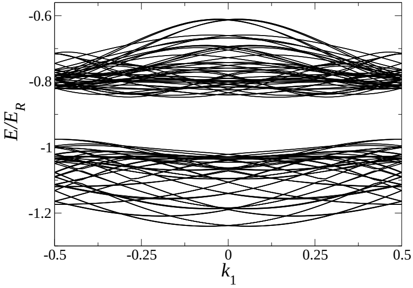




Three Dimensions

This nearly-free electron viewpoint leads to a general method to construct $\mathbb{Z}_2$ non-trivial bands in 3D. [Benjamin Béri & NRC, arXiv:1101.5610]

e.g. $\delta \to \delta_0 \cos(\boldsymbol{\kappa}_3 \cdot \mathbf{r})$ $c_{12} \to c_{12} + \delta_0(\mu + c_{13} + c_{23})$



$\mathcal{V} = 0.9 E_{\rm R}$
$\delta_0 = 1, \mu = -0.4$

$\Rightarrow$3D insulator with relativistic (Dirac) 2D surface states.

Summary

- Simple forms of optical dressing lead to "optical flux lattices": periodic magnetic flux with high mean density, $n_\phi \sim 1/\lambda^2$.
- The low energy bands are analogous to the lowest Landau level of a charged particle in a uniform magnetic field.
- These could lead to fractional quantum Hall states, with sizeable energy scales.
- The approach can be generalized to generate $\mathbb{Z}_2$ nontrivial bandstructures in 2D and 3D.