The Half-Filled Landau Level

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Celebration for Bert Halperin's 75th
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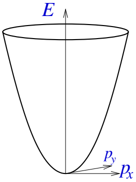
Chong Wang, Bert Halperin & Ady Stern

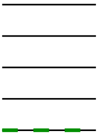
[C. Wang, NRC, B. I. Halperin & A. Stern, arXiv:1701.00007]

Outline

The Half-Filled Landau Level

2D band + magnetic field B^{ext} (no spin!)





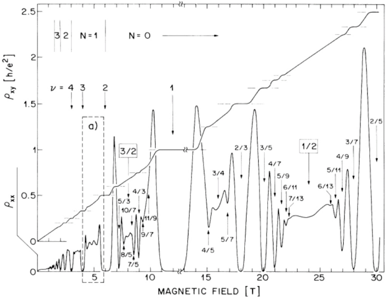
- degenerate Landau levels,

$$n_\phi = eB^{\mathrm{ext}}/h$$

- partial filling $\nu = n/n_\phi$

$\Rightarrow$strong correlations

- $\nu = 1/2$ compressible



[Willett, Eisenstein, Störmer, Tsui, Gossard, English, 1987]

"Unquantized Quantum Hall Effect"

Composite Fermion Liquid

[Halperin, Lee & Read, 1993]

$$\hat{H} = \sum_i \frac{(\boldsymbol{p}_i + e\boldsymbol{A}_i^{\text{ext}})^2}{2m_e} + \sum_{i<j} V(\boldsymbol{r}_i - \boldsymbol{r}_j)$$

$$\to \sum_i \frac{(\boldsymbol{p}_i + e\boldsymbol{a}_i + e\boldsymbol{A}_i^{\text{ext}})^2}{2m_e} + \sum_{i<j} V(\boldsymbol{r}_i - \boldsymbol{r}_j)$$

fermion = electron + 2 flux of auxiliary magnetic field

$$\boldsymbol{\nabla}_i \times \boldsymbol{a}_i = -2\frac{h}{e}\sum_{j\neq i}\delta(\boldsymbol{r}_i - \boldsymbol{r}_j)$$

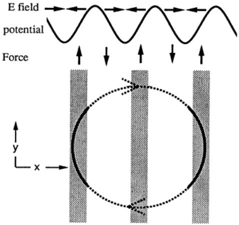
mean-field theory, $B^{\text{eff}} = B^{\text{ext}} - 2\frac{h}{e}n$

- $B^{\text{eff}} = 0$ at $\nu = 1/2$ $\Rightarrow$Fermi surface (m^*, F_ℓ,...)
- $\nu = \frac{p}{2p+1}$ $\Rightarrow$CFs fill p LLs (FQH states)

Composite Fermion Liquid

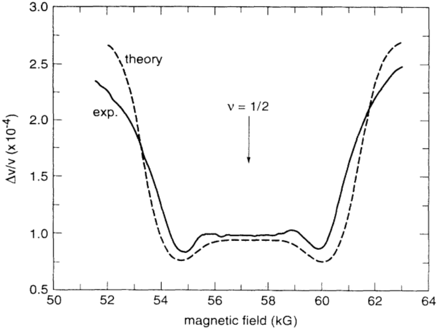
Emergent lengthscale, $R_c^{\mathrm{eff}} = \dfrac{k_F \hbar}{e|B^{\mathrm{eff}}|}$ [Halperin, Lee & Read, 1993]

$\Rightarrow$ geometric resonances



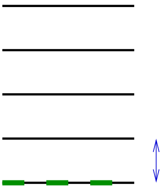
$qR_c^{\mathrm{eff}} \sim \pi(i + 1/4)$

surface acoustic waves



[Willett, West & Pfeiffer, 1993]

Particle-Hole Symmetry



$\hbar\omega_c = \hbar \frac{eB^{\text{ext}}}{m_e}$

$m_e \to 0 \Rightarrow$ no Landau level mixing

electrons at ν $\Leftrightarrow$ holes at $1-\nu$

particle-hole symmetry at $\nu = 1/2$ [two-body forces]

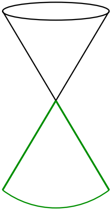
HLR theory is not explicitly particle-hole symmetric...

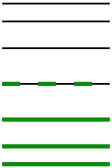
...is it even *incompatible* with this symmetry?

Dirac Composite Fermion Liquid [D.T. Son, 2015]

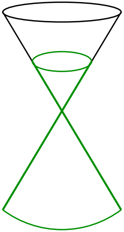
[Son; Wang & Senthil; Metlitski &Vishwanath; Mross, Alicea & Motrunich;...]

2D Dirac cone + magnetic field B^{ext}





+ duality arguments ⇒



$n = 0$ LL as for parabolic band
intrinsic particle-hole symmetry

Dirac sea of CFs
(p-h ⇒time-reversal)

fermionic duality in 2+1D [Son; Wang & Senthil; Metlitski &Vishwanath; Mross, Alicea & Motrunich; Seiber, Senthil, Wang & Witten; Karch & Tong; Murugan & Nastase]

cf. bosonic duality in 3D [Dasgupta & Halperin, 1981]

Dirac CFs are *vortices* of original theory: $n_{\mathrm{CF}}^{\mathrm{Dirac}} = \frac{1}{2}\frac{eB^{\mathrm{ext}}}{h}$

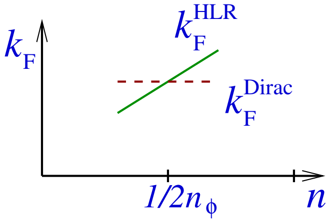
(1) HLR vs Dirac: Cyclotron radius

$$R_c^{\text{eff}} \equiv \frac{k_F \hbar}{e|B^{\text{eff}}|} = \frac{k_F}{2\pi|n - n_\phi/2|}$$

At fixed $n_\phi = \frac{eB^{\text{ext}}}{h}$ particle-hole symmetry relates n to $n_\phi - n$

HLR: $n_{\text{CF}}^{\text{HLR}} = n$

Dirac: $n_{\text{CF}}^{\text{Dirac}} = \frac{1}{2}\frac{eB^{\text{ext}}}{h}$

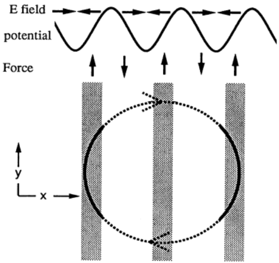


HLR appears inconsistent with particle-hole symmetry

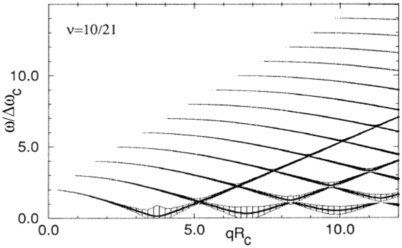
but, we need to calculate a physical observable...

[C. Wang, NRC, B.I. Halperin & A. Stern, arXiv:1701.00007]

HLR CFL: Finite wavevector response



[Simon & Halperin, PRB 1993]



Semiclassical analysis within RPA...

...minima of magnetoroton spectrum *close to* $qR_c^{\text{eff}} \simeq \pi(i + 1/4)$

▷ Keeping corrections to order $(n - n_\phi/2)^2$, the *same* theory shows particle-hole symmetry [C. Wang, NRC, B.I. Halperin & A. Stern, arXiv:1701.00007]

(2) HLR vs Dirac: Hall conductivity

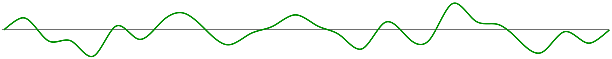
Particle-hole symmetry $\Rightarrow \sigma_{xy} = \frac{1}{2}\frac{e^2}{h}$ [Kivelson, Lee, Krotov & Gan, 1997]

For HLR: $\hat{\sigma} = \left[\hat{\rho}^{\mathrm{CF}} + \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} \frac{h}{e^2}\right]^{-1}$

particle-hole symmetry would require $\sigma_{xy}^{\mathrm{CF}} = -\frac{1}{2}\frac{e^2}{h}$

But... ...for particle-hole symmetric disorder...

$V(r)$



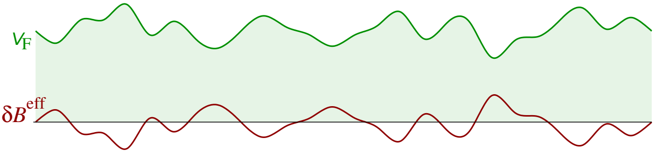
$$\delta n^{\mathrm{CF}}(\boldsymbol{r}) = -\chi V(\boldsymbol{r}) \;, \quad \delta B^{\mathrm{eff}}(\boldsymbol{r}) = -2\frac{h}{e}\delta n^{\mathrm{CF}}(\boldsymbol{r})$$

CFs experience vanishing average magnetic field $\Rightarrow \sigma_{xy}^{\mathrm{CF}} = 0$ (?)

Hall conductivity of HLR state

[C. Wang, NRC, B. I. Halperin & A. Stern, arXiv:1701.00007]

Correlation of scalar potential with magnetic field



CFs move faster where $\delta B^{\text{eff}} < 0$ $\Rightarrow$ larger Lorentz force ($\sigma_{xy}^{\text{CF}} < 0$)

▷ Full calculation* $\Rightarrow \sigma_{xy}^{\text{CF}} = -\frac{1}{2}\frac{e^2}{h}$

as required for particle-hole symmetry

[*classical "Kubo" formula, or quantum calculation of "side-jump" scattering in Born approx.]

Discussion

- HLR theory makes predictions for low-frequency and long-wavelength observables that are consistent with particle-hole symmetry.

- It is far from obvious that these results should emerge from the HLR theory. (Certainly not a convenient route!)

- A stronger feature: these results hold even in the absence of microscopic particle-hole symmetry (e.g. $m_e \neq 0$, or $\nu = 1/4$)

$\Rightarrow$ *emergent* particle-hole symmetry

- Open issues: suppressed $2k_F$ backscattering, Hall viscosity...

[Geraedts et al., Science 2016; Levin & Wen, arXiv 2016;...]

Summary

- The HLR theory is compatible with particle-hole symmetry: HLR and Dirac theories describe the same phase of matter.
- Renewed interest has raised new questions and created links to other areas of physics (exotic phases on surfaces of topological insulators, quantum spin liquids, fermionic dualities...).
- The "unquantized quantum Hall effect" remains an inspiring and intriguing area of investigation.