Density Waves and Supersolidity in Rapidly Rotating Atomic Fermi Gases

Nigel Cooper
T.C.M. Group, Cavendish Laboratory,
University of Cambridge

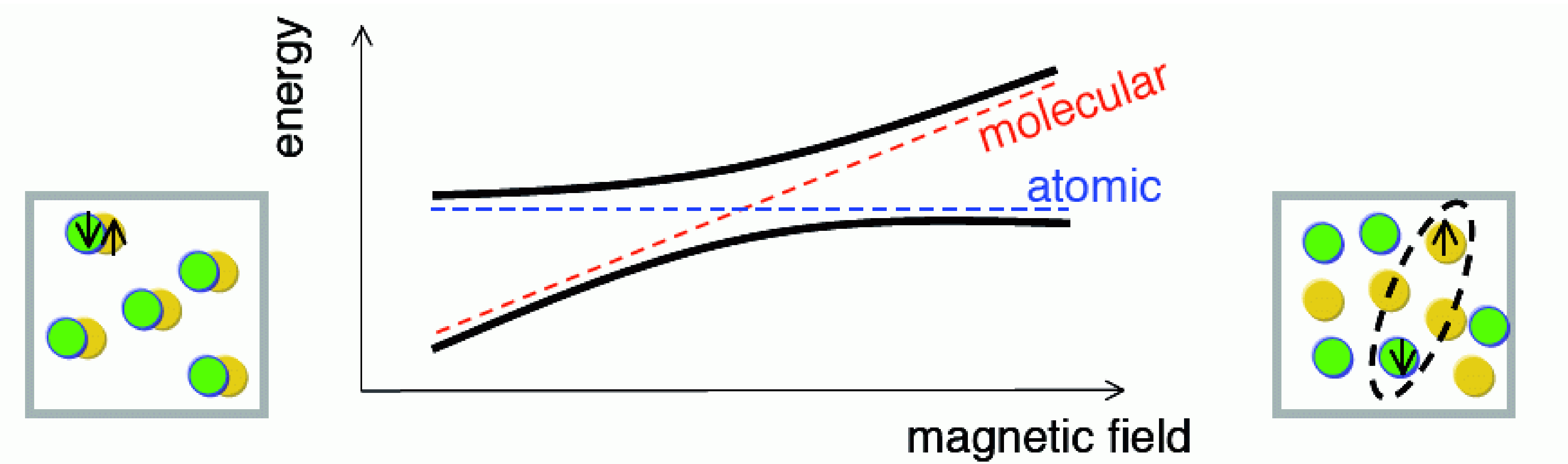
Quantum Gases Conference, Paris, 30 June 2007.

Gunnar Möller and NRC, arxiv/0704.3859

EPSRC
Engineering and Physical Sciences Research Council

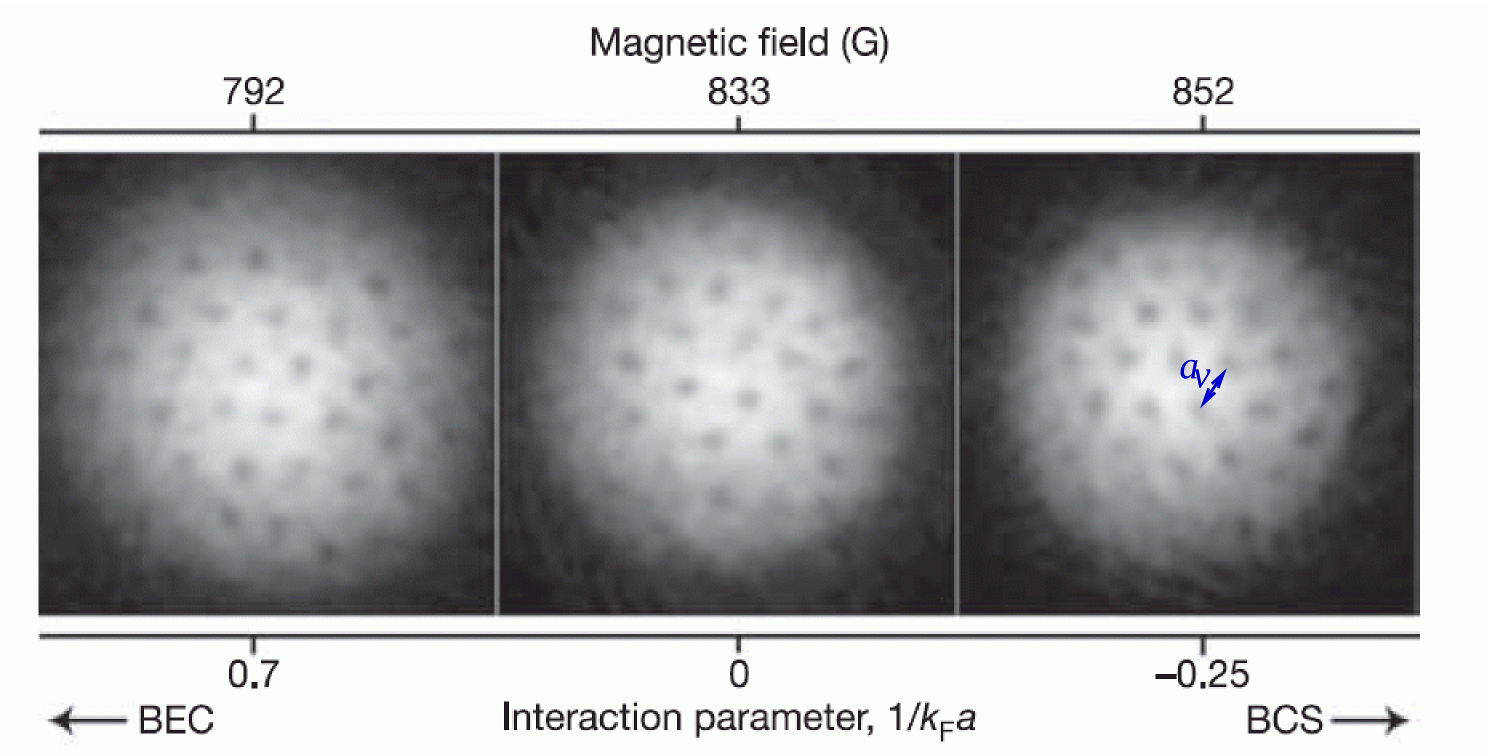
BEC-BCS crossover

Two hyperfine states of the same fermionic atom: $\uparrow$, $\downarrow$



BEC of tightly-bound bosonic molecules $\overset{\text{smooth crossover}}{\Longleftrightarrow}$ BCS state fermionic atoms.

What is the effect of rapid rotation on the balanced Fermi gas?



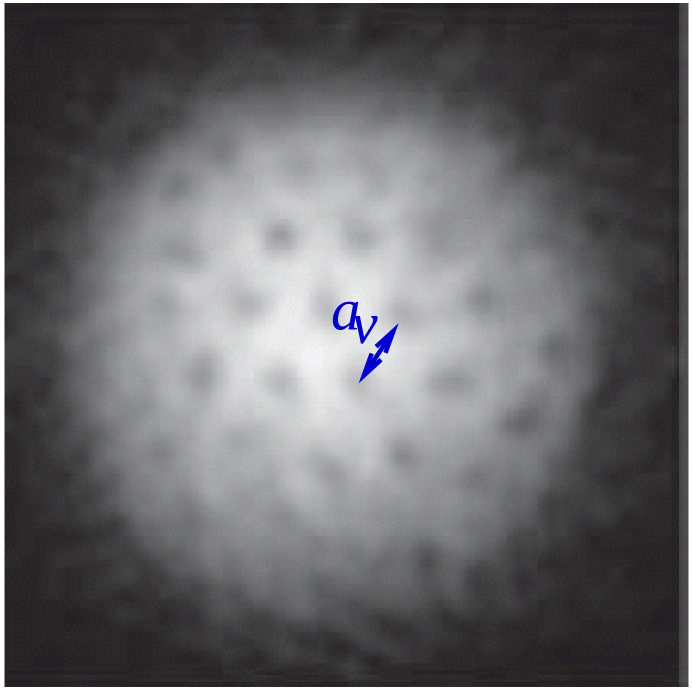
Vortex density
$\frac{2M\Omega}{\hbar} \sim \frac{1}{a_V^2}$

[Zwierlein, Abo-Shaeer, Schirotzek, Schunck & Ketterle, Nature **435**, 1047-1051 (2005)]

What sets the "upper critical rotation rate" on the BCS side?

- Semiclassical approximation (small λ_F): $\hbar\Omega \gtrsim \frac{\Delta^2}{\epsilon_F}$ [Gorkov, JETP **9**, 1364 (1959)]

[BCS gap $\Delta \sim \epsilon_F\ e^{\frac{\pi}{2k_F a}}$]



Vortex density, $\frac{2M\Omega}{\hbar} \sim \frac{1}{a_V^2}$

superfluid velocity, $\sim \frac{\hbar}{Ma_V}$

K.E. per pair, $\sim \frac{1}{2}M\left(\frac{\hbar}{Ma_V}\right)^2 \sim \hbar\Omega$

- "High-field" superconductivity [*e.g.* Rasolt & Tešanović, RMP **64**, 709 (1992)]

- Beyond mean-field theory.

Overview

- What is the fate of the BCS superfluid at rapid rotation?

– BCS Mean Field Theory: “high field” superconductivity

– Beyond MFT: Density waves and supersolidity

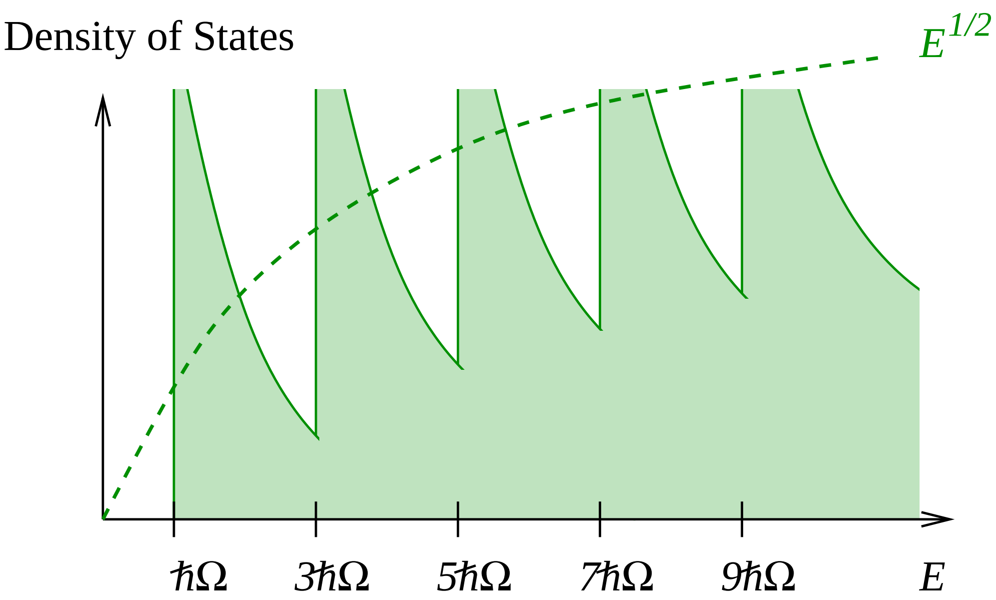
(• Strong-coupling: Evolution across the Feshbach resonance)

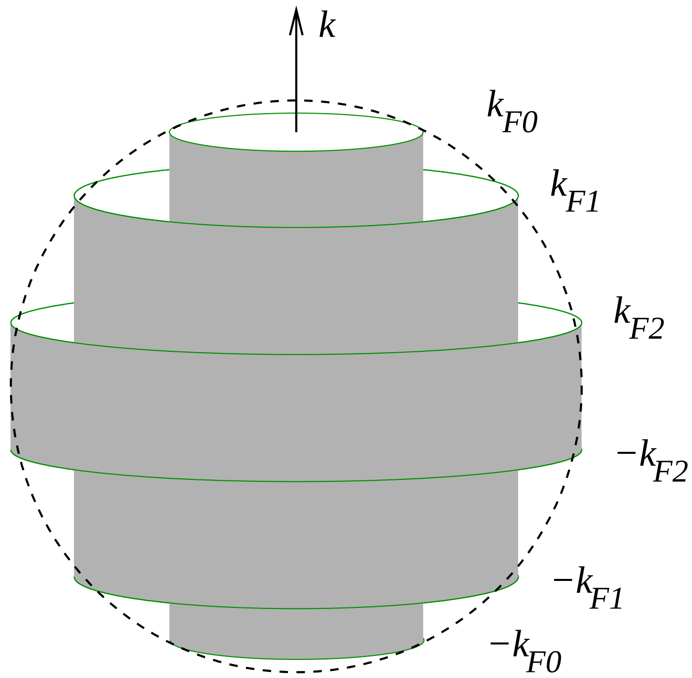
Rapidly Rotating Fermi Gas

Uniform 3D regime: $\Omega \simeq \omega_\perp \gg \omega_\parallel$

$$\epsilon_{nk} = (n + 1/2)2\hbar\Omega + \frac{\hbar^2 k^2}{2m}$$

quasi-1D Fermi surface for each LL.





What is the groundstate for (weak) attractive s-wave interactions?

BCS Mean Field Theory for a Rotating Fermi Gas

Related studies:

[Veillette, Sheehy, Radzihovsky & Gurarie, PRL **97**, 250401 (2006); Zhai & Ho, PRL **97**, 180414 (2006)]

Linearized Gap equation

$$\frac{1}{-a_s} = \hbar\Omega \sum_{n,n'=0}^{\infty} \binom{n+n'}{n} \frac{1}{2^{n+n'}} \int \frac{dk}{2\pi} \left[\frac{\tanh \frac{\epsilon_{nk}-\mu}{2k_BT} + \tanh \frac{\epsilon_{n'k}-\mu}{2k_BT}}{\epsilon_{nk} + \epsilon_{n'k'} - 2\mu} - \frac{2}{\epsilon_{nk}+\epsilon_{n'k'}} \right]$$

$k_BT_c \ll \hbar\Omega$

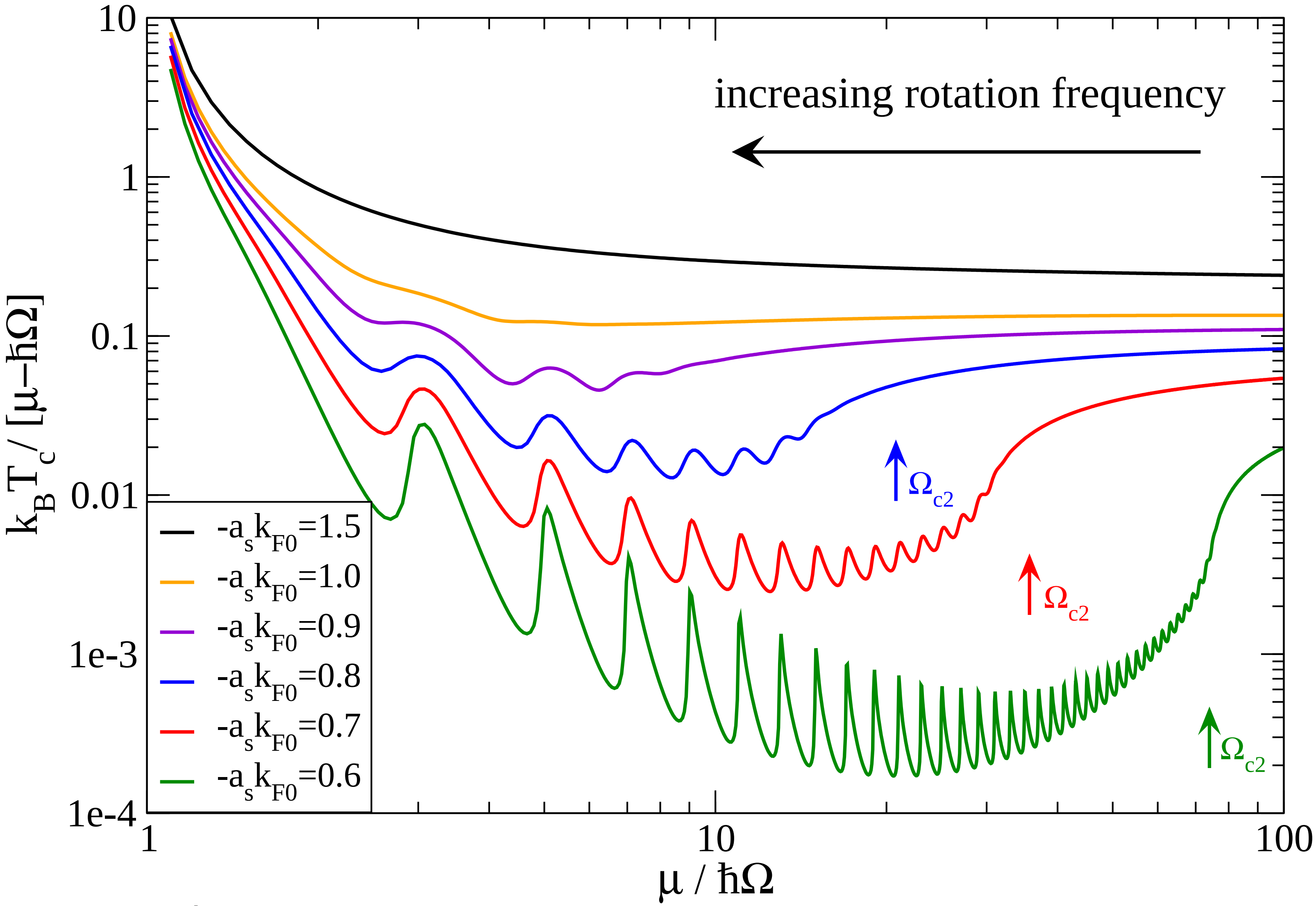
[Gunnar Möller & NRC, arxiv/0704.3859]

$$T_c \sim \eta \frac{\hbar\Omega}{k_B} \exp\left\{ \frac{2\pi}{a_s k_{F0}} G(\eta)^{-1} \right\} \quad ; \quad G(\eta) \equiv \frac{1}{\eta} \sum_{n=0}^{n_{\max}} \frac{(2n)!}{(2^n n!)^2} \left(1 - \frac{n}{\eta}\right)^{-\frac{1}{2}} .$$

$$\eta \equiv (\mu - \hbar\Omega)/(2\hbar\Omega)$$

Numerical Solution

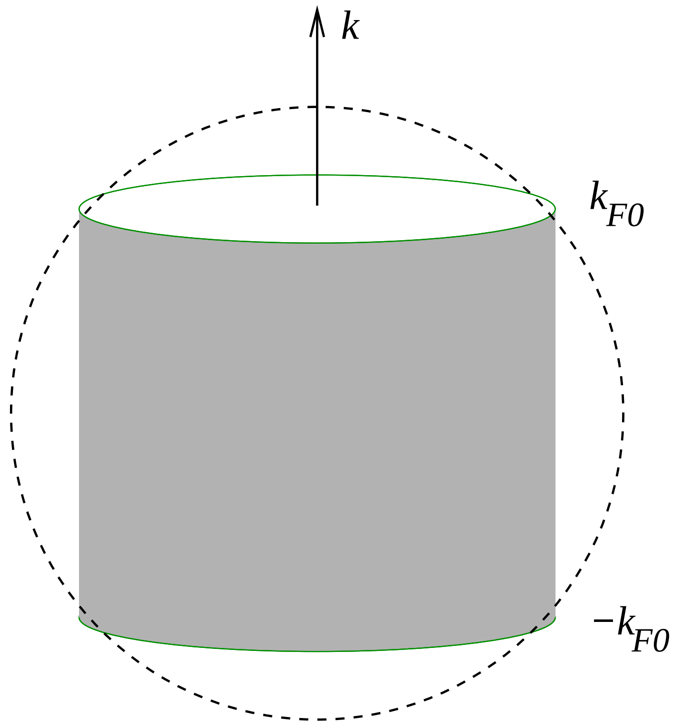
[Gunnar Möller & NRC, arxiv/0704.3859]



- T_c never vanishes.
- T_c/μ is an *increasing* function of $\hbar\Omega/\mu$ for atoms in the lowest Landau level.

Lowest Landau Level limit, $n = 0$

[Gunnar Möller & NRC, arxiv/0704.3859]



$\epsilon_{0k} = \hbar\Omega + \frac{\hbar^2 k^2}{2m}$

quasi-1D Fermi surface

$\frac{2m\Omega}{\hbar} \equiv \frac{1}{2\pi\ell^2}$ states per unit area.

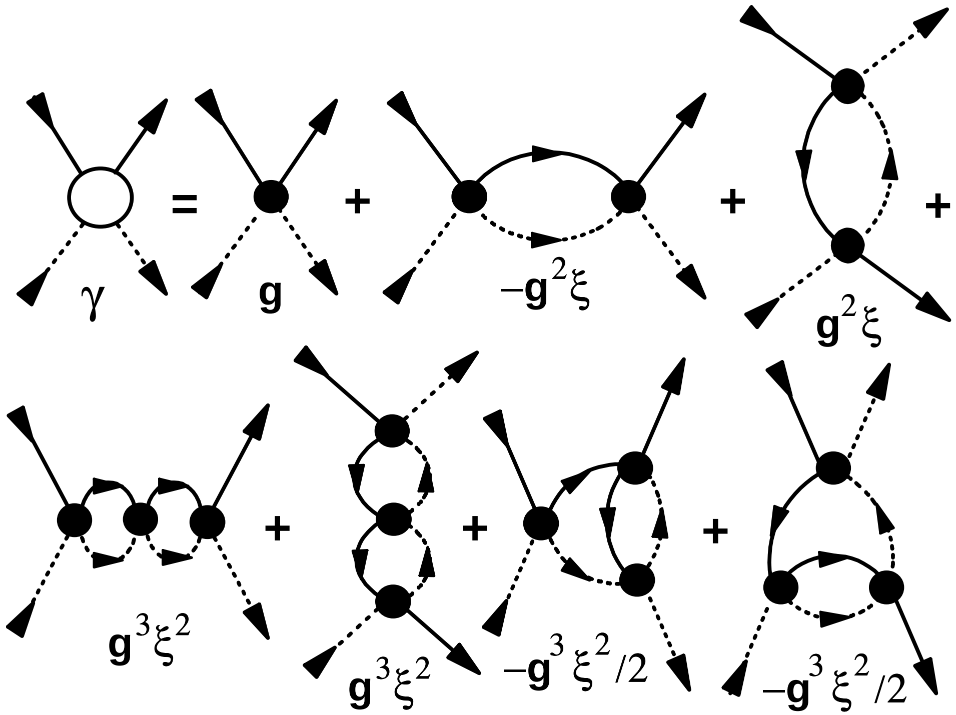
Attractive interactions $\Rightarrow$Fermi-surface instabilities at low T.

Within mean field theory BCS and CDW instabilities both occur at the *same* T_c.

Parquet diagrams

[Brazovskii, Zh. Eksp. Teor. Phys. **61**, 2401 (1972);

Yakovenko, PRB **47**, 8851 (1993)]



$$\xi = \frac{|g|}{(2\pi)^3\hbar v_F \ell^2} \ln\left(\frac{\epsilon_F}{k_B T}\right)$$

$$g = 4\pi\hbar^2 a_s/M$$

RG equations for *spin-degenerate* case [Gunnar Möller & NRC, arxiv/0704.3859]

$$\frac{d\gamma_1}{d\xi} = -2\,\gamma_1 * \gamma_1 + 2\,\gamma_1 * \gamma_2 - 2\,\gamma_1 \otimes \gamma_2$$

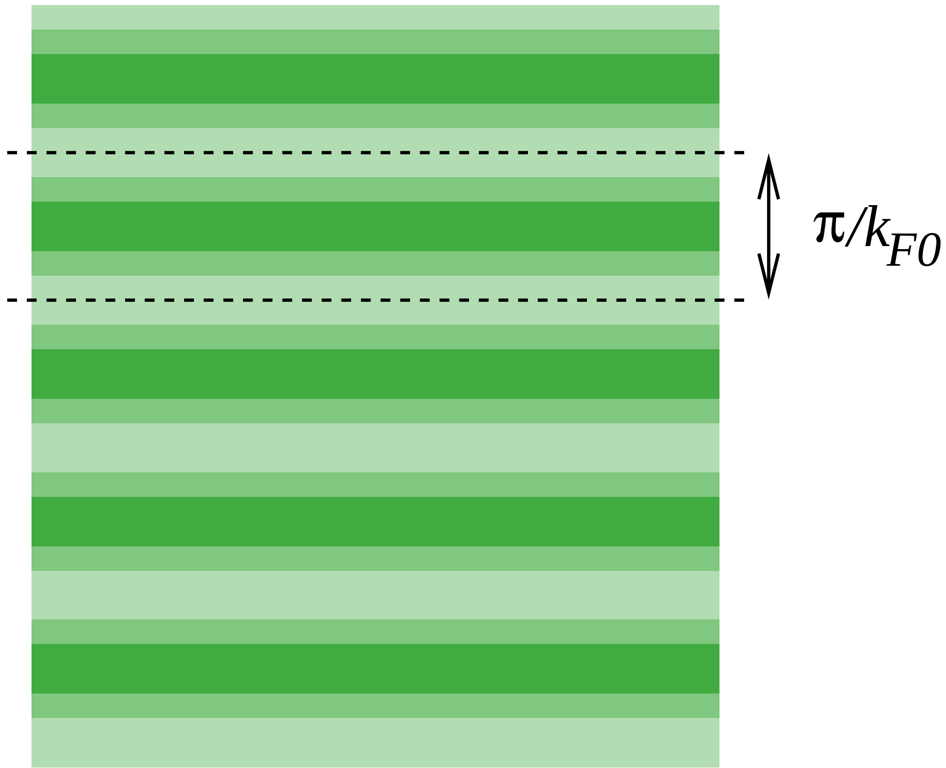
$$\frac{d\gamma_2}{d\xi} = 2\,\gamma_2 * \gamma_2 - \gamma_1 \otimes \gamma_1 - \gamma_2 \otimes \gamma_2$$

Numerical solution

[Gunnar Möller & NRC, arxiv/0704.3859]

Attractive contact interactions, $n = 0 \Rightarrow$ CDW at $\xi_c = 0.726/(2\pi)$.

$$T_c^{CDW} \sim \frac{\epsilon_F}{k_B} \exp\left(-\frac{(2\pi)^3 \hbar v_F \ell^2}{|g|}\, \xi_c\right)$$



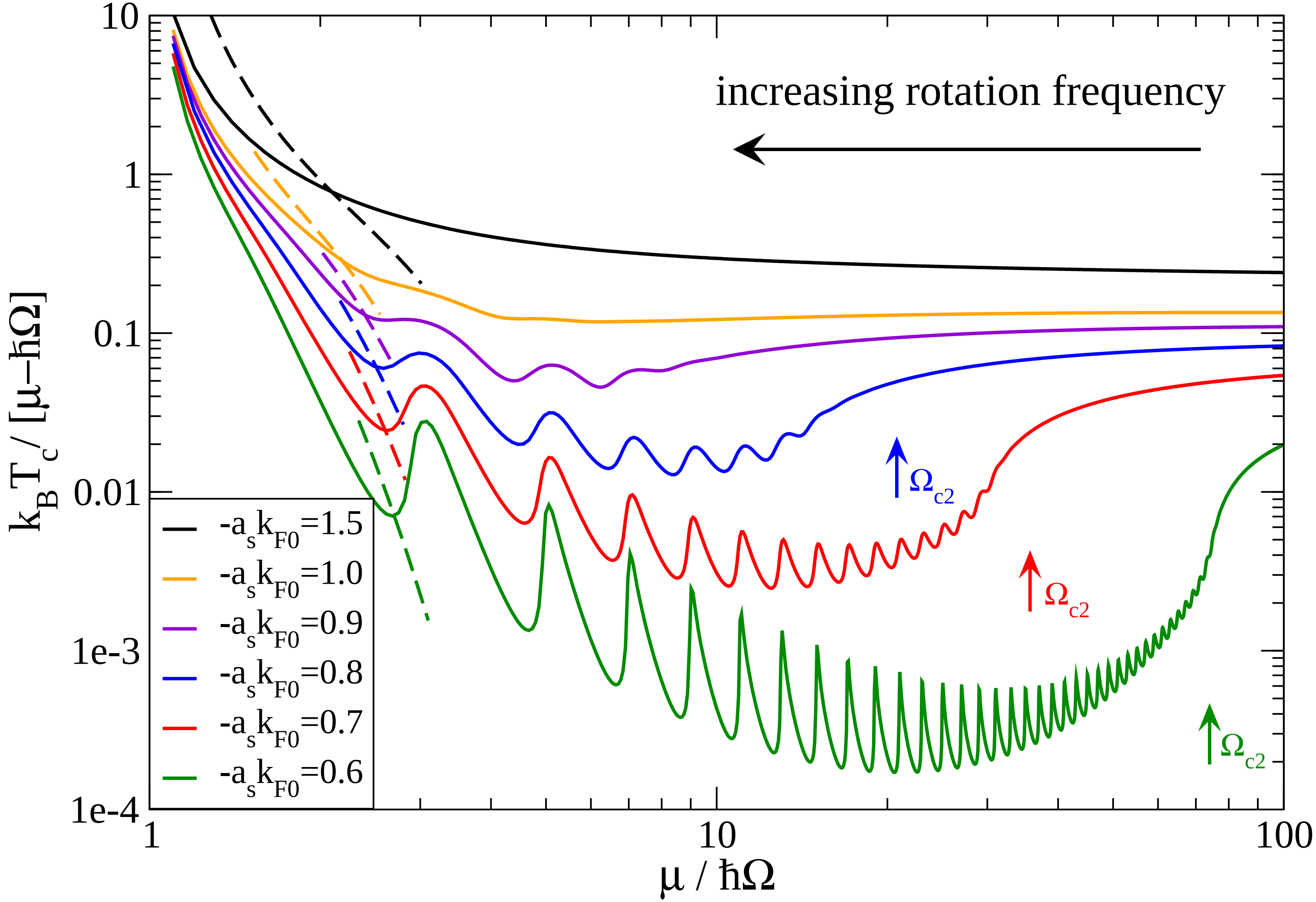
Density per layer, $n_{2\mathrm{d}} \equiv n \frac{\pi}{k_{F0}} = \frac{4m\Omega}{\hbar}$

$i.e.$ filling factor, $\nu_a \equiv n_{2\mathrm{d}}(2\pi\ell^2) = 2$

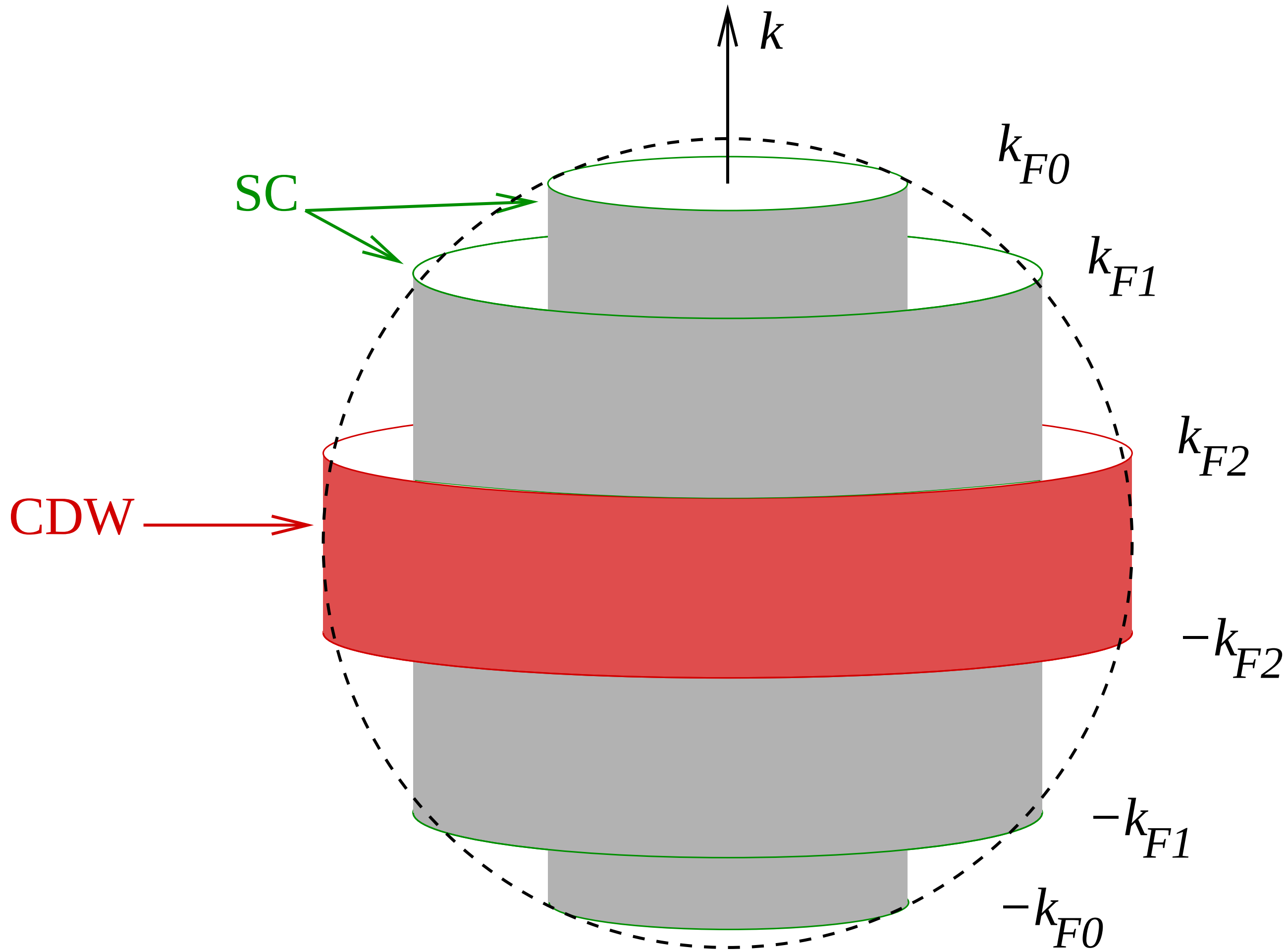
A CDW of $\nu_a = 2$ states, with uniform 2D density in each layer

Comparison with BCS theory

[Gunnar Möller & NRC, arxiv/0704.3859]



- The "upper critical rotation rate" is set by the transition into a CDW.



- Coexistence of CDW in the upper LL and SC in lower LLs: supersolidity.

Implications for Experiment

- 3D regime: $\omega_{\|} \lesssim \omega_{\perp} \Rightarrow a_{\|} \gtrsim a_{\perp}$.

- Breakdown of semiclassical BCS theory: $\hbar\Omega \gtrsim \frac{\Delta^2}{\epsilon_F}$

- Lowest Landau level: $\mu, k_B T \lesssim 2\hbar\Omega$

c.f. bosons [Schweikhard, Coddington, Engels, Mogendorff & Cornell, PRL **92**, 040404 (2004)]

Low filling factors more easily accessible than for (weakly-interacting) bosons.

[Antezza, Cozzini & Stringari PRA **75**, 053609 (2007)]

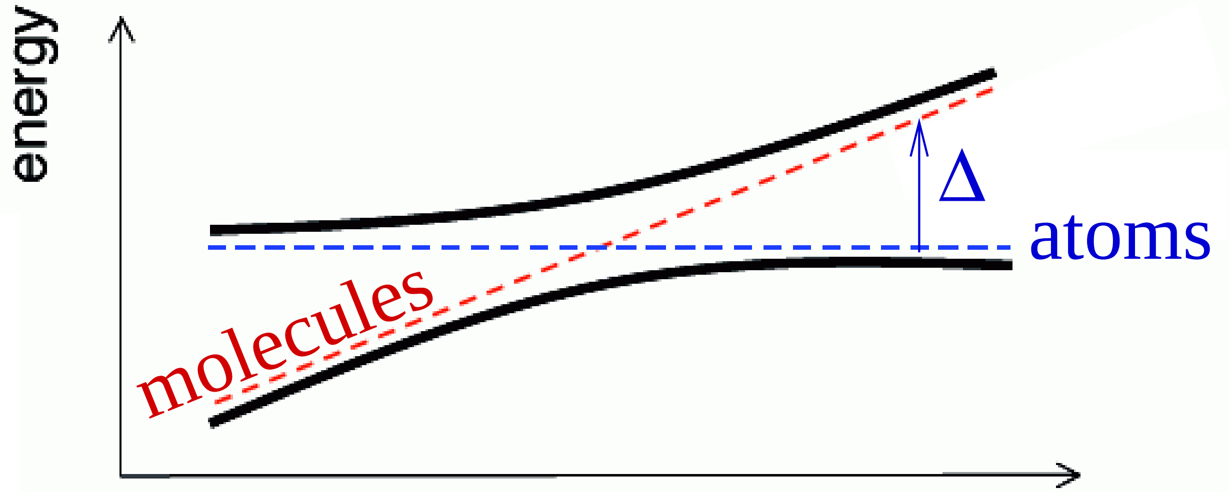
▷ Spontaneous formation of density wave order, with period $\lambda_n = \frac{\pi}{k_{Fn}} \geq \frac{\pi}{2} a_{\perp}$.

2D Fermi Gas with strong interactions

[Duncan Haldane, Ed Rezayi]

Strong attractive interactions $\Rightarrow$ atoms form small bosonic molecules.

$$n_m = \frac{n_a}{2} \quad , \quad \ell_m = \sqrt{\frac{\hbar}{2(2m)\Omega}} = \frac{\ell_a}{\sqrt{2}} \quad \Rightarrow \quad \nu_m = \frac{1}{4}\nu_a$$



$\nu_m = 1/2$ *bosonic* Laughlin liquid $\quad \Longleftrightarrow \quad$ $\nu_a = 2$ fermion QH liquid

These two states are separated by a *phase transition*

Summary

- Rapidly rotating cold atomic Fermi gases offer the opportunity to explore superconductivity beyond the conventional semiclassical regime.

- BCS theory applied to a rapidly rotating gas shows a non-zero T_c for any rotation rate: "high-field" superconductivity.

- The upper critical rotation rate is determined by physics that goes beyond mean-field theory.

- Superfluidity is destroyed by the appearance of CDW order.
– At intermediate rotation rates CDW and SC coexist (supersolidity);
– In the lowest Landau level the groundstate is a CDW of layers of $\nu_a = 2$.

- In 2D, crossing the Feshbach resonance drives a $phase\ transition$ between a $\nu_a = 2$ state of atoms and a $\nu_m = 1/2$ Laughlin state of molecules.