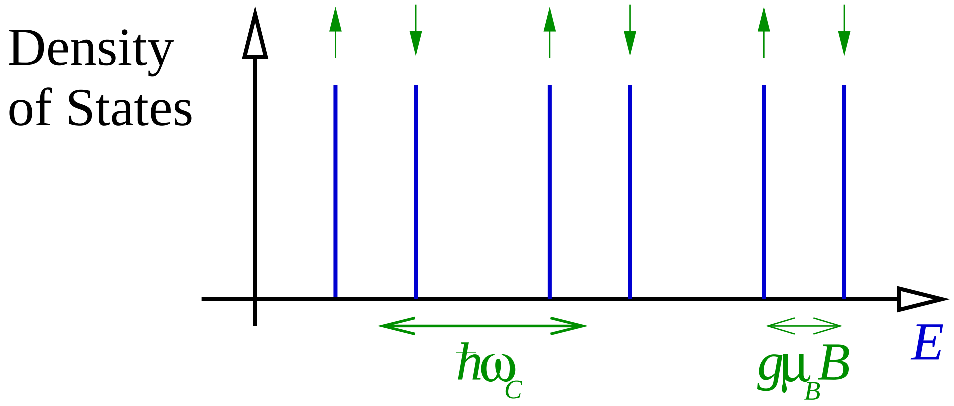
Skyrmion Physics in Quantum Hall Ferromagnets

Nigel Cooper
T.C.M. Group, Cavendish Laboratory, Cambridge

CMMP 97, Exeter, 18 December 1997

What is a Quantum Hall Ferromagnet?

Two-dimensional electron gas, in magnetic field B.



"Filling fraction" $\nu \equiv n(2\pi\ell^2)$ $\quad (\ell \equiv \sqrt{\hbar/eB})$

$+$ Interactions

Typically (GaAs) $e^2/\epsilon\ell \gg g\mu_B B$
$\Rightarrow$ Interactions are important for the spin-order

"Ferromagnetic" if the groundstate is spin-polarized for $g \to 0$. [Consider $e^2/\epsilon\ell \ll \hbar\omega_c$]

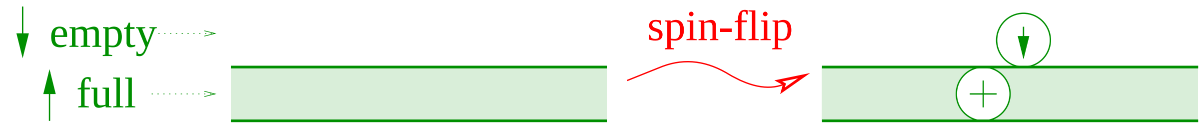
$\nu = 1, 3, 5\ldots$ are ferromagnetic (also $1/3, 1/5\ldots$)
[Lee & Kane, 1990; Sondhi et al., 1993; MacDonald et al., 1996]

Interactions can have dramatic effects on the excitations.

Spin-Waves and Quasiparticles

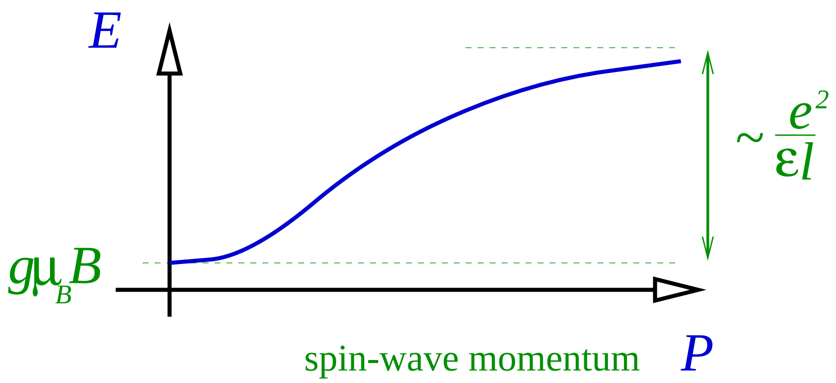
[Bychkov, Iordanskii & Eliashberg, 1981; Kallin & Halperin, 1984]

Ferromagnet $\Longrightarrow$ low-energy spin-flip excitations

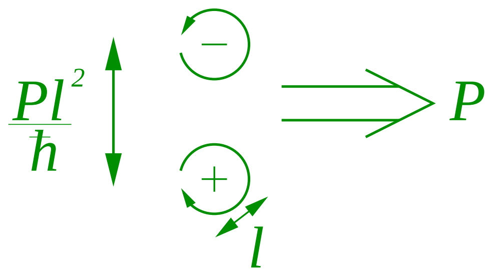


$\hbar\omega_c \gg e^2/\epsilon\ell \Longrightarrow$ spin-wave = magnetic exciton

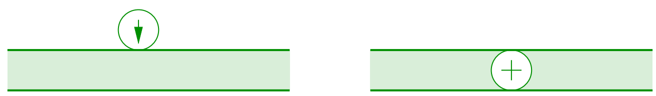
Dispersion Relation $(\nu = 1)$ [Lerner & Lozovik, 1980]



Semiclassical Picture:



$P\ell/\hbar \to \infty \Longrightarrow$ quasielectron/quasihole pair.



Are these the lowest-energy charged excitations?

A Continuum Field-Theory

For long-wavelength deformations, the ferromagnet may be described by a local magnetization $\vec{n}(\vec{r})$

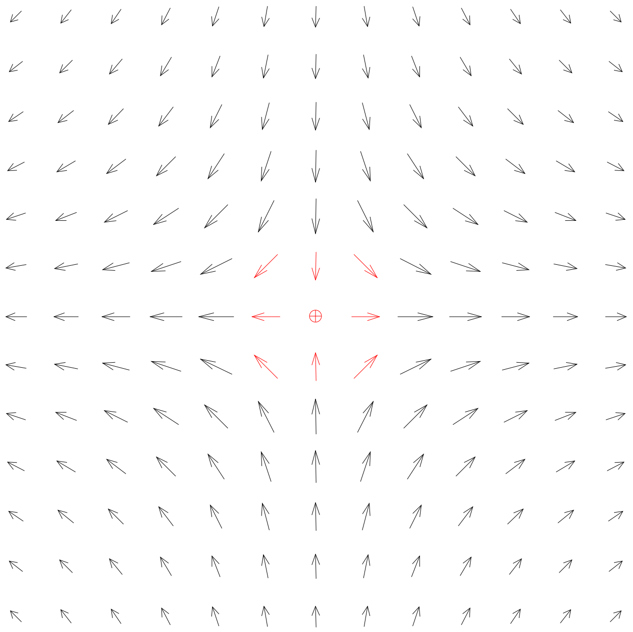
$$E = \frac{1}{2}\rho_s \int d^2\vec{r}\, (\vec{\nabla}\vec{n})^2 \quad (+\, g\mu_B B N)$$

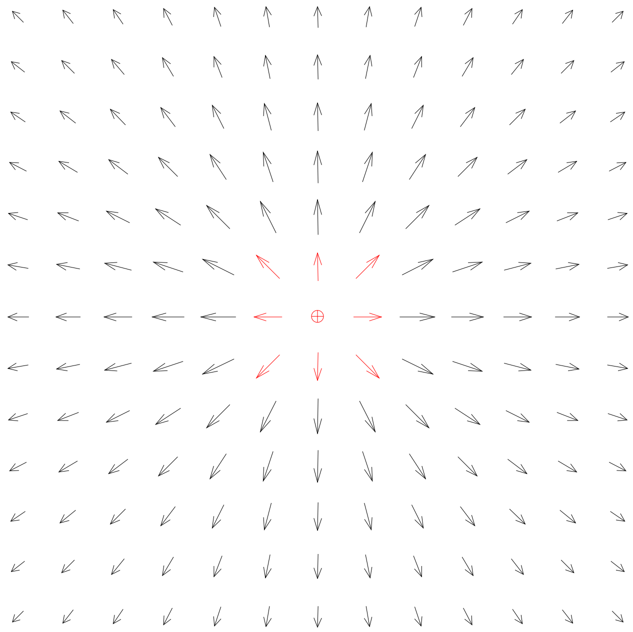
Topological density: $q(\vec{r}) \equiv \frac{1}{8\pi}\epsilon_{ijk}\epsilon_{\alpha\beta}n_i\nabla_\alpha n_j\nabla_\beta n_k$
Topological index: $Q \equiv \int d^2\vec{r}\; q(\vec{r}) =$ integer.

Lowest-Energy Configurations [Belavin & Polyakov, 1975]

$Q = 0 \rightarrow$ Uniform Ferromagnet

$Q = 1 \rightarrow$ "Skyrmion" $Q = -1 \rightarrow$ "anti-Skyrmion"





Quantum Hall Skyrmions

Topological density $\Longleftrightarrow$ (physical) charge density

Skyrmion carries a charge $-|e|$ $\quad(-\nu|e|)$

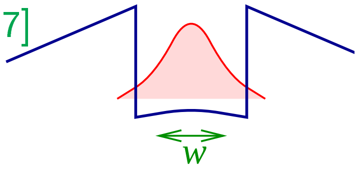
anti-Skyrmion carries a charge $+|e|$ $\quad(+\nu|e|)$

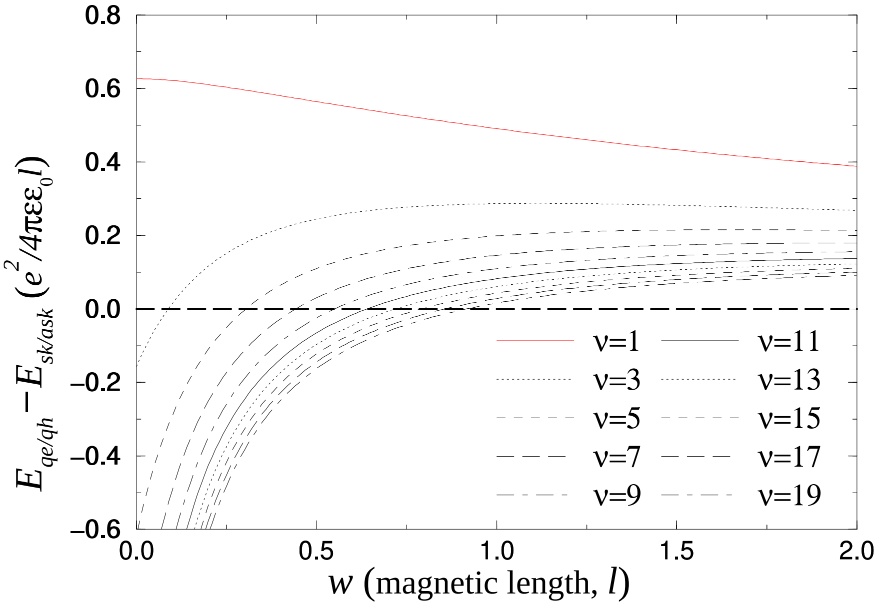
[Lee & Kane, 1990; Sondhi et al., 1993]

How do their energies compare to those of the quasiparticles? $(g\mu_B B = 0)$

Coulomb interactions: Skyrmions have lower energies at $\nu = 1$, but *not* at $\nu = 3, 5, \ldots$ [Sondhi et al, 1993; Wu & Sondhi, 1995]

In fact, the answer depends sensitively on details of the short-range interactions. [N.R.C., 1997]





(FQHE Skyrmions thought to be favoured for Coulomb interactions)

Is there a continuum analogue of the spin-wave?

Dynamical Solitary Waves

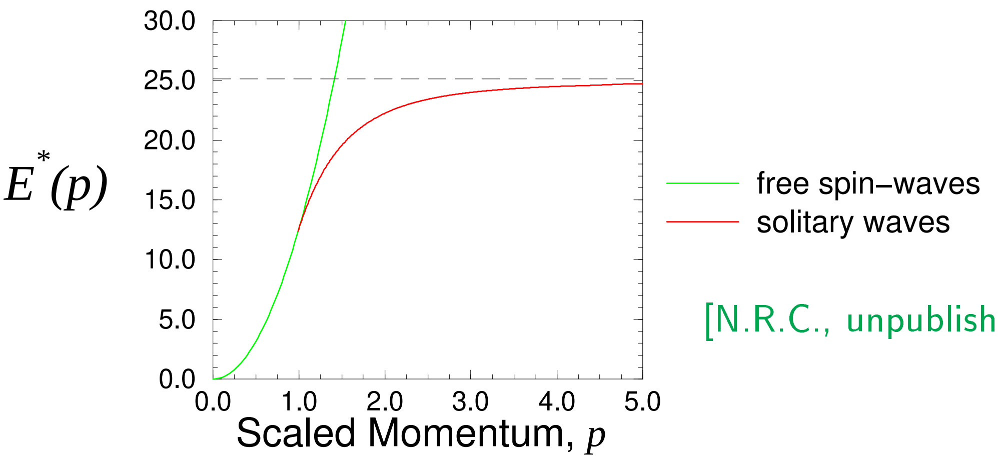
The long-wavelength dynamics of the ferromagnet are described by the Landau-Lifshitz equation:

$$\dot{\vec{n}} = 4\pi\vec{n} \times \nabla^2\vec{n} \qquad (\hbar = \ell = \rho_s = 1)$$

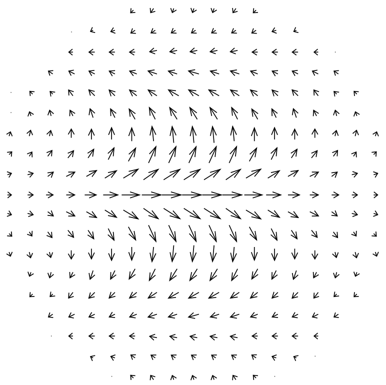
We search for neutral excitations $(Q = 0)$, by determining the minimum energy $E^*(\vec{P}, N)$ for given momentum $\vec{P}$, and number of spin-flips N.

$\Rightarrow$ a branch of travelling-waves of velocity $\vec{v} = \frac{\partial E^*}{\partial \vec{P}}$.

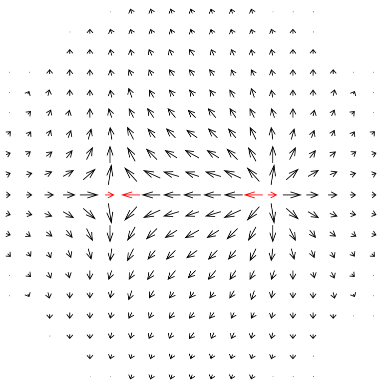
Scale invariance: $E^*(\vec{P}, N) = E^*(p \equiv |\vec{P}|/\sqrt{N})$



[N.R.C., unpublished]



spin-wave droplet



Skyrmion/anti-Skyrmion pair

Summary

- Electron-electron interactions can have dramatic effects on the spin-order of quantum Hall systems ($g\mu_B B \ll e^2/\epsilon\ell$).

- The groundstates at certain integer, and fractional filling fractions are ferromagnetic.

- The addition, or removal, of charge can lead to the creation of a Skyrmion, or anti-Skyrmion, involving large local spin-depolarization.

- "Spin-wave droplets" and "Skyrmion/anti-Skyrmion pairs" describe solitary waves of the ferromagnet.