Vortex Liquids in Atomic Bose Condensates and the Fractional Quantum Hall Effect

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[N.K. Wilkin & J.M.F. Gunn, cond-mat/9906282; N.R.C. & N.K. Wilkin, cond-mat/9907498; R.A. Smith, unwritten]

Overview

- Magneto-Optically-Confined Atomic Bose Condensates

- Multi-Vortex states: Mean Field Theory
  Exact Diagonalisations

- Comparison to the FQHE: Composite Fermions

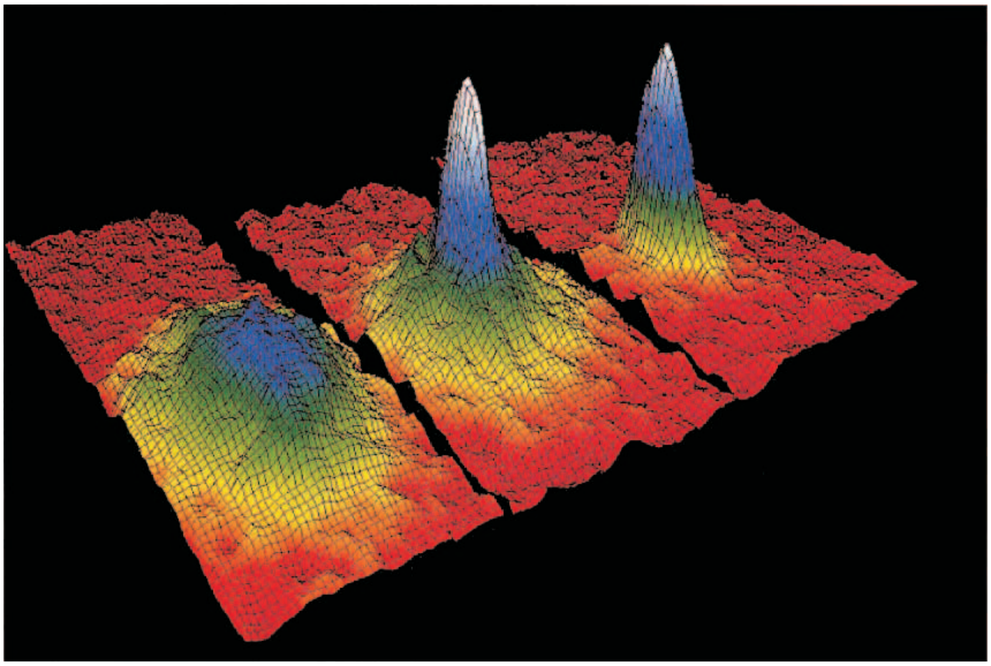
- Summary

Magneto-Optically-Confined BECs

Bose-Einstein Condensation

$$k_B T_c \simeq \frac{\hbar^2}{m}\bar{n}^{2/3} \sim 100\text{nK}$$

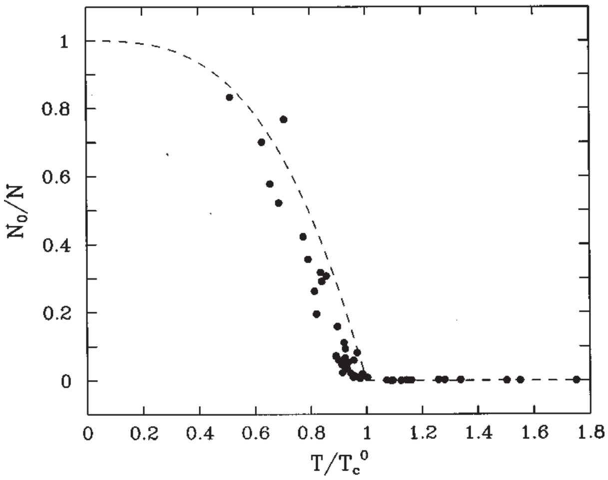
^{87}Rb (JILA, 1995)
^{7}Li (Rice, 1995)
^{23}Na (MIT, 1995)
^{1}H (MIT, 1998)



[Anderson *et. al.* [JILA], Science **269**, 198 (1995).]

Condensed Fraction

[Ensher *et. al.* [JILA], PRL **77**, 4984 (1996).]

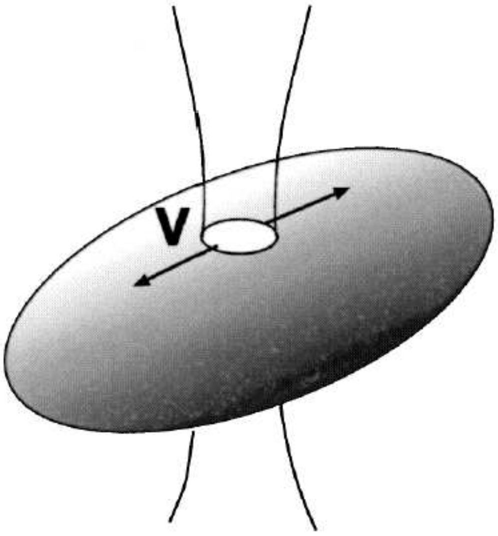


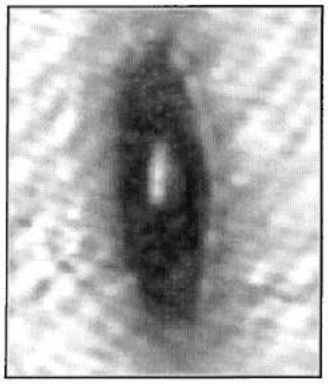
Condensate fraction as a function of T/T_c^0.

$$N \sim 40,000$$

Superfluidity?

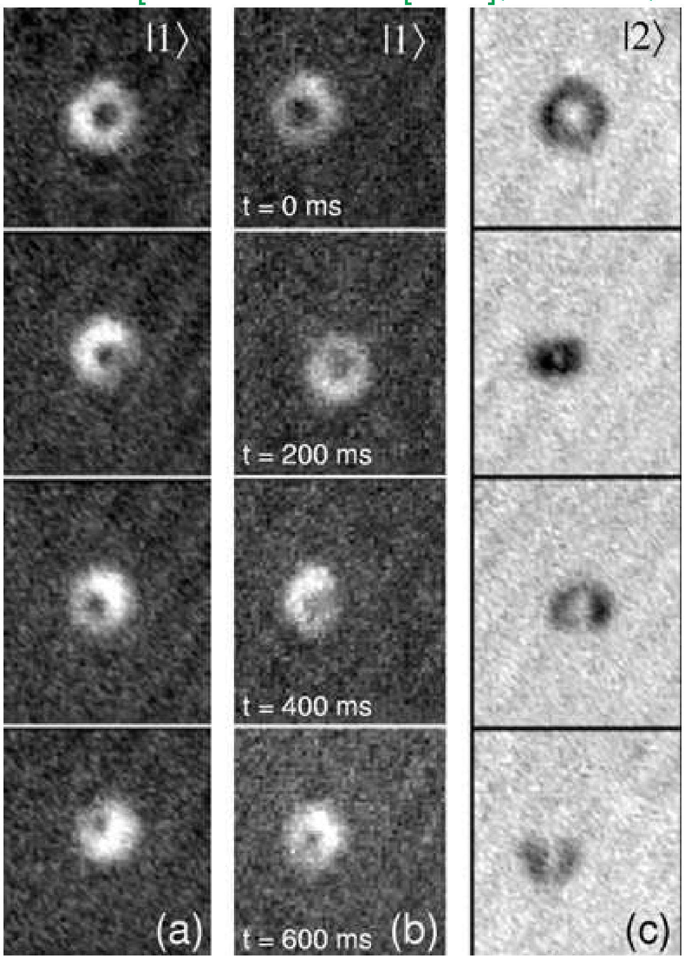
[Raman *et. al* [MIT], PRL, **83**, 2503 (1999)]





Vortices

[Matthews *et al.*[JILA], PRL **83**, 2498 (1999)]



What's special?

Tunable effective interactions

s-wave scattering length, a.

atom	a
^{87}Rb	5.77nm
^{23}Na	2.75nm
^{7}Li	-1.45nm
^{1}H	0.07nm

Weakly interacting:

$$\begin{aligned} a &\sim 1\text{nm} \\ \bar{a} &\sim 10\text{nm} \end{aligned}$$

$\Rightarrow$ Condensed fraction is large: $\frac{N_{\rm out}}{N} \sim \left(\frac{a}{\bar{a}}\right)^{3/2} \sim 1\%$

Healing length

$\xi \sim \frac{1}{\sqrt{\bar{n}a}} \sim 500\text{nm}$ $\qquad a_{ho} \sim 1000\text{nm}$

$\Rightarrow$ vortex cores are very large.

> What are the (multi)-vortex states of a BEC when $\xi \gg a_{ho}$?

$[L \neq 0,\ T = 0]$

$\xi \gg a_{ho}$ = The Lowest Landau Level

$$
\begin{aligned}
H &= \sum_i \left\{ \frac{p_i^2}{2m} + \frac{1}{2} m \left[\omega_\parallel^2 (x^2 + y^2) + \omega_z^2 z^2 \right] \right\} \\
&\quad + \frac{\eta}{2} \sum_{i \neq j} \delta(\vec{r_i} - \vec{r_j})
\end{aligned}
$$

Single-Particle Spectrum

$E = \hbar\omega_z (n_z + 1/2) + \hbar\omega_\parallel (2n_r + |m| + 1)$

+ Interactions

$\eta \ll \hbar\omega_{\parallel,z} \Rightarrow n_z = n_r = 0$
$L = \sum_i m_i$ conserved $\Rightarrow m \geq 0$

$L \neq 0 \Rightarrow$ Degenerate perturbation theory in the "Lowest Landau Level" [Wilkin, Gunn & Smith, PRL **77**, 4984 (1998)]

$$\langle x, y | m \rangle \propto z^m e^{-|z|^2/2} \qquad [z \equiv (x + iy)/a_{ho}]$$

$$a_{ho} \equiv \sqrt{\hbar / m\omega_\parallel}$$

+ Rotations

$$E = E_o + V(L) + \hbar(\omega_\parallel - \omega) L$$

Gross-Pitaevskii Mean-Field Theory

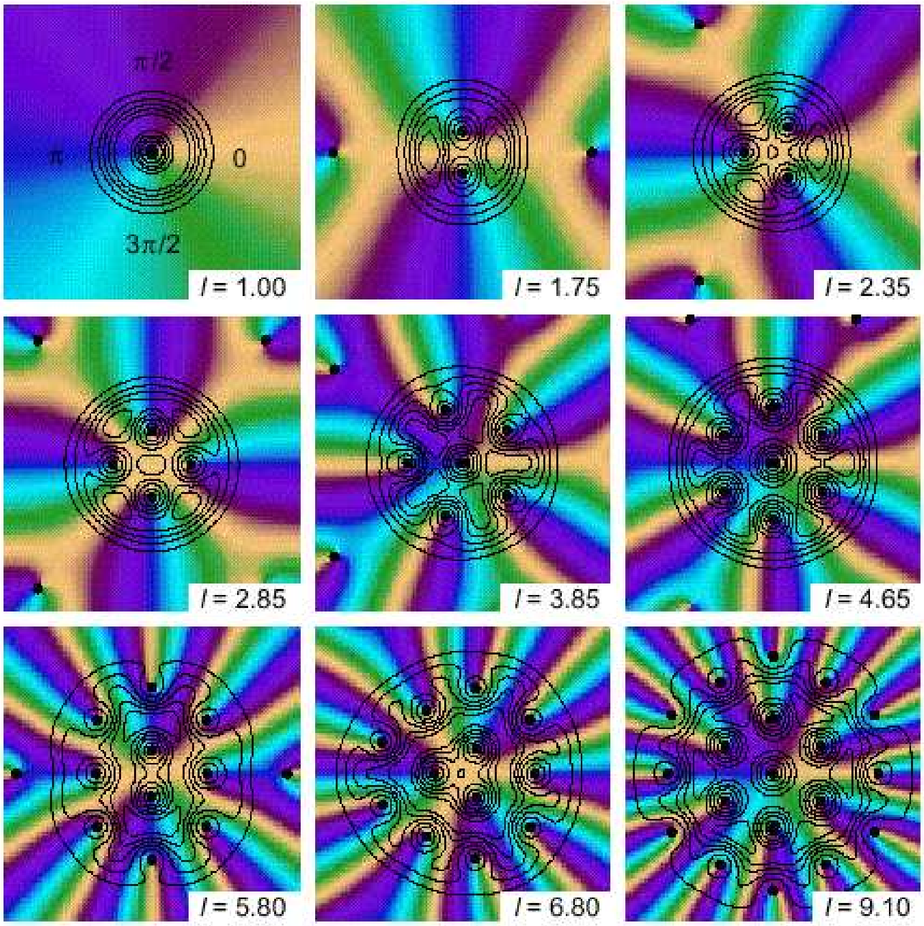
$$\Psi(\{r_i\}) = \prod_i \phi(\vec{r_i})$$

Minimise the energy functional

$$\int d^3\vec{r} \left\{ \frac{\hbar^2}{2m} |\nabla\phi|^2 + V(\vec{r})\,|\phi|^2 + \frac{Ng}{2}|\phi|^4 \right\}$$

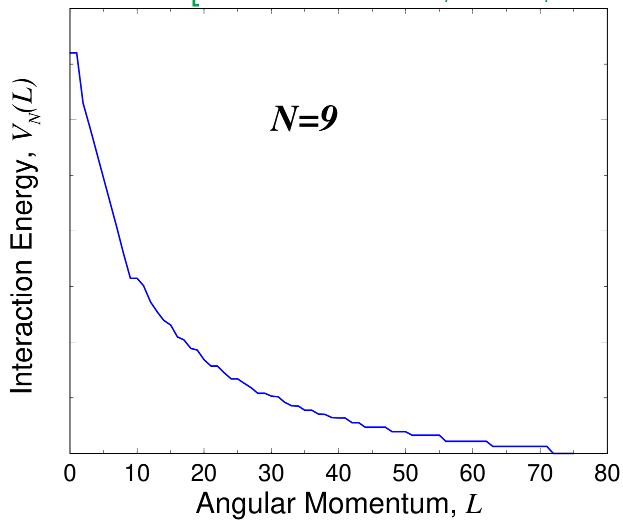
$Ng \ll \hbar\omega_{\|}, \hbar\omega_z \Longrightarrow \phi(\vec{r}) = \sum_{m=0}^{\infty} c_m \langle x, y|m\rangle \phi_0(z).$

Numerical results: [Butts & Rohksar, Nature **397**, 327 (1999)]



Exact Diagonalizations

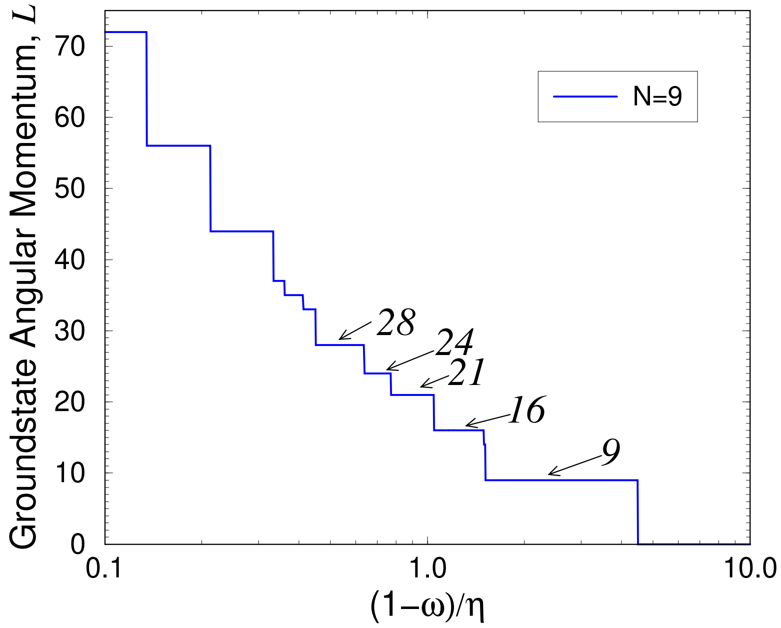
[Wilkin & Gunn, 1999; NRC & Wilkin, 1999]



$L \leq N$ Exact wavefunctions [RA Smith, unpublished]

$$\Psi \propto \prod_i (z_i - z_c)$$

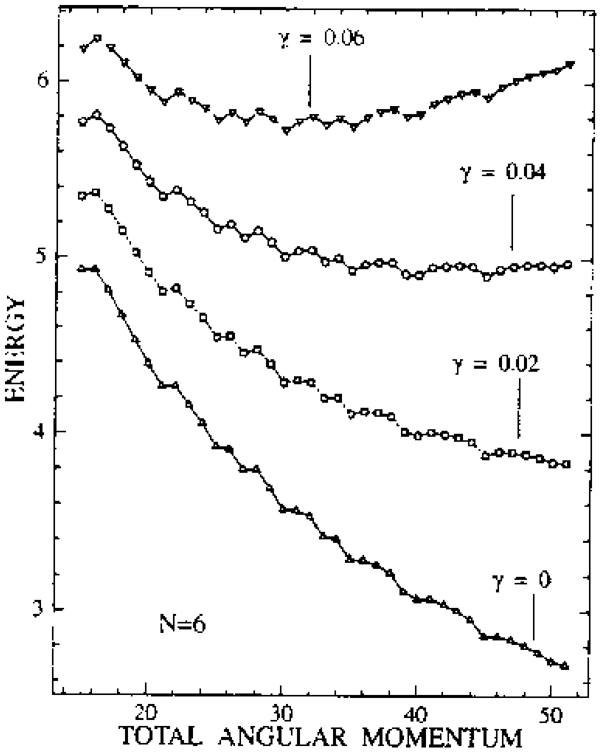
$L > N$ Complicated sequence of stable states



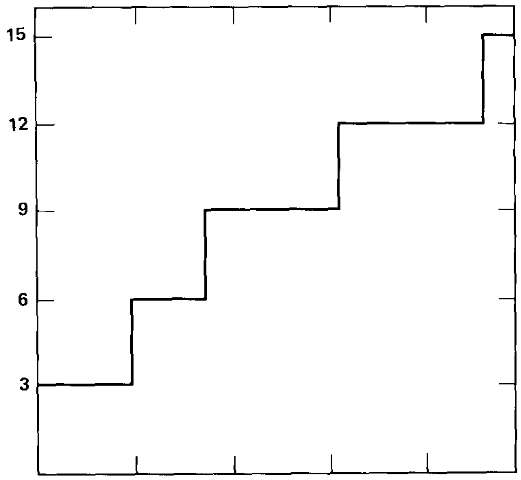
No simple interpretation in terms of classical configurations of vortices. [Wilkin & Gunn, cond-mat/9906282]

Quantum Dots: "Magic" angular momenta

Quantum Dot: Interacting 2D electrons in strong perpendicular magnetic field ⇒ lowest Landau level + harmonic confinement potential $[V = \gamma(x^2 + y^2)]$.



[Seki *et. al.*, JPSJ, (1996).]

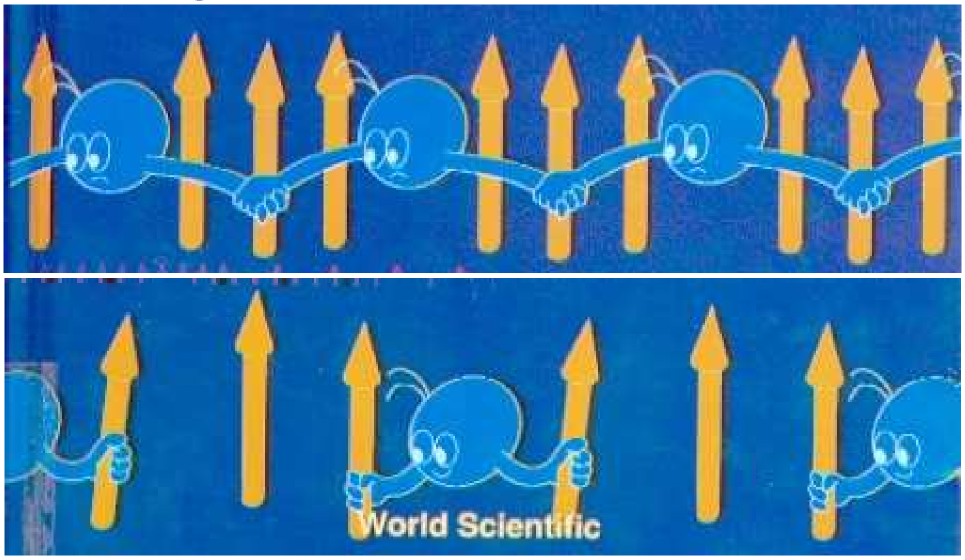


[Laughlin, (1983).]

Here: Formally equivalent: fermions → bosons (δ-function interactions)

Composite Fermions in Fractional Quantum Hall Systems

Interacting electrons in strong magnetic field $\Rightarrow$ non-interacting *composite fermions*.



Composite fermion = bound state of an electron with a *vortex* of the many-body wavefunction.

Jain's formulation:

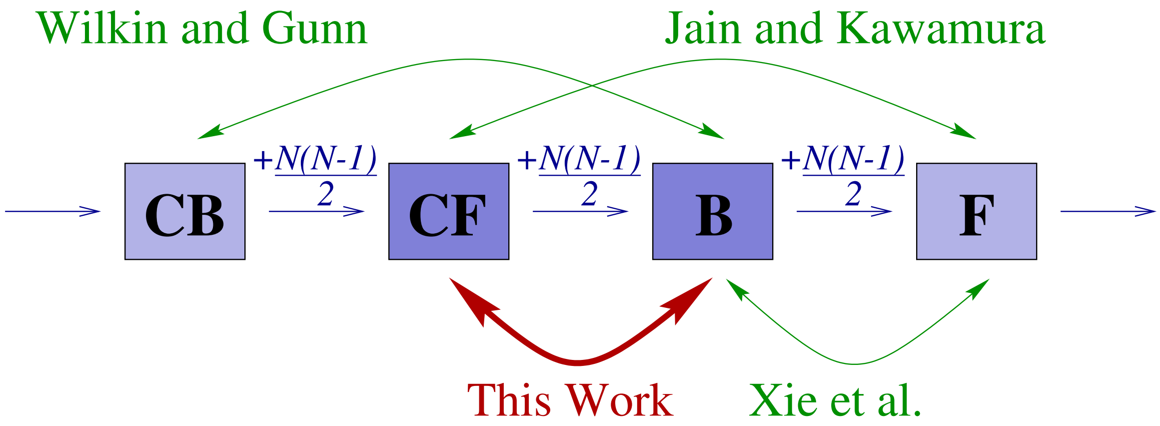
$$\psi_{\mathrm{E}}(\{z_i\}) = \mathcal{P} \prod_{i<j} (z_i - z_j)^{2n} \psi_{\mathrm{CF}}(\{z_i, z_i^*\})$$

• Pushes electrons apart $[L = L_{\mathrm{CF}} + N(N-1)]$;
• Pauli principle for $\psi_{\mathrm{CF}}(\{r_i\})$ also helps;
• the CFs behave as non-interacting fermions with a *kinetic energy*: the groundstate is (typically) non-degenerate.

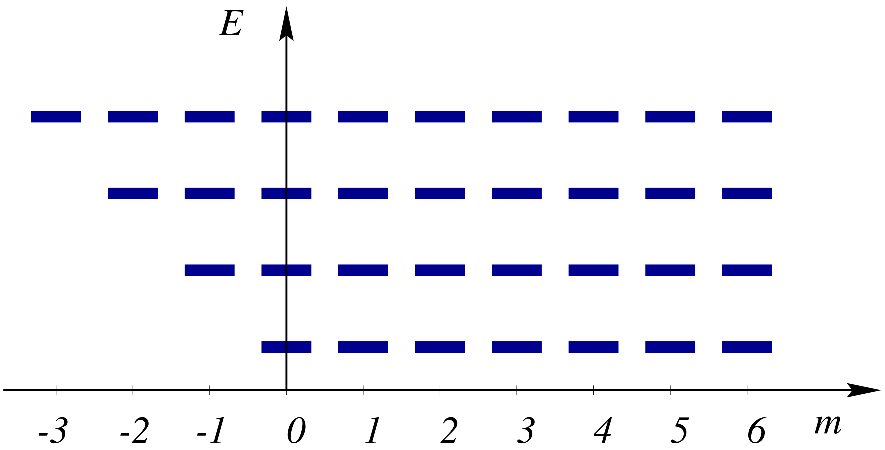
Composite Fermions for Rotating Bose Condensates

$$\psi_{\mathrm{B}}(\{z_i\}) = \mathcal{P}\prod_{i<j}(z_i - z_j)^1 \psi_{\mathrm{CF}}(\{z_i, z_i^*\})$$

$$L = L_{\mathrm{CF}} + \frac{N(N-1)}{2}$$

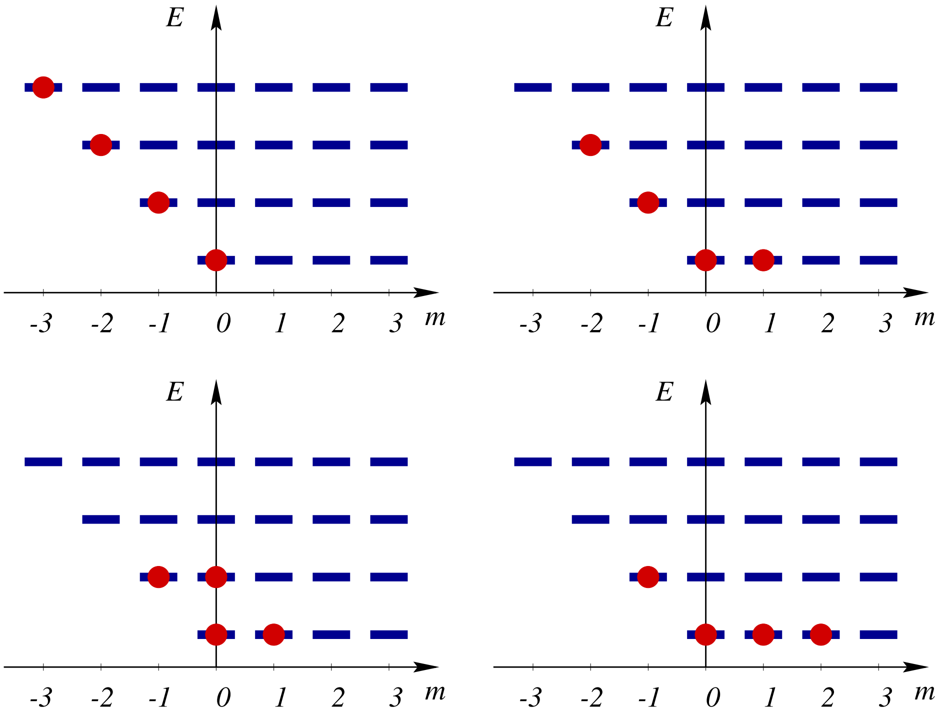


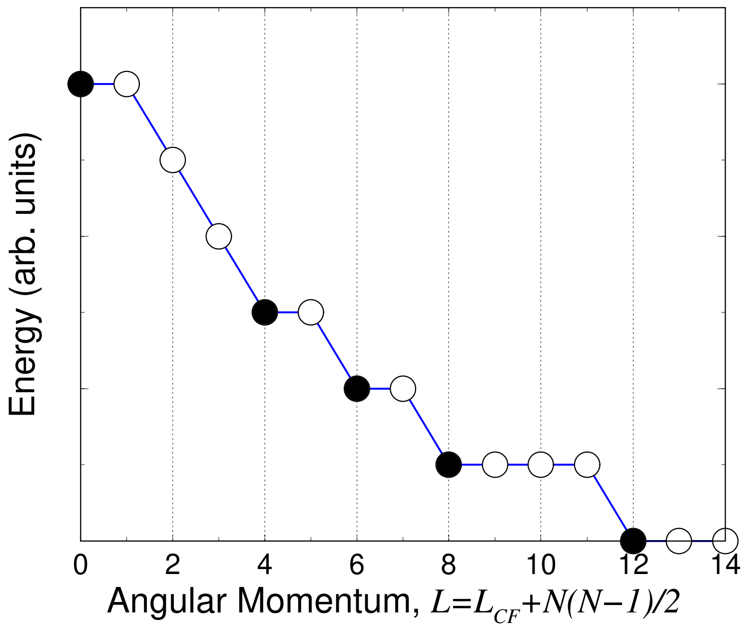
CF Basis States



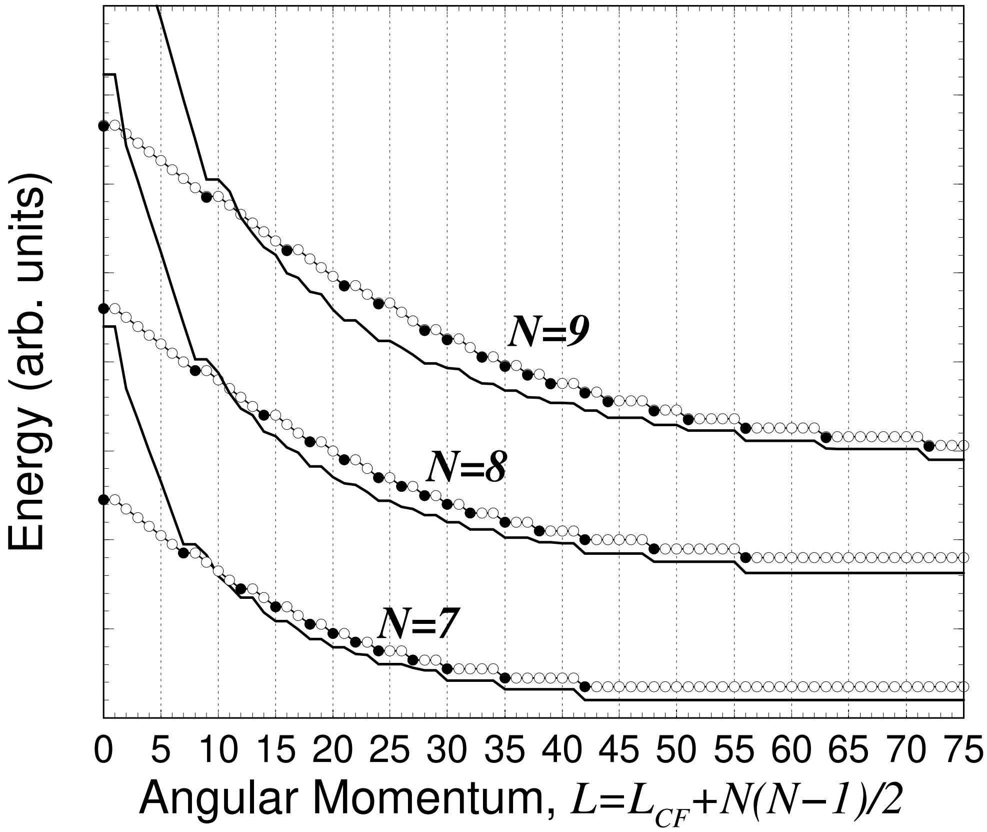
Non-interacting CFs: Stable States

$L = L_{\rm CF} + \frac{N(N-1)}{2}$





Stable states: $L \in L^*_{N=4} = \{0, 4, 6, 8, 12\}$



$e.g.$ N=7

CF states: $L^*_{N=7} = \{0, 7, 12, 15, 18, 20, 22, 24, 27, 30, 35, 42\}$

Exact results: $L = 0, 7, 12, 15, 18, 24, 30, 35, 42$

$\Rightarrow$ the CF model correctly predicts the stable angular momenta of the boson spectrum.

Wavefunctions Excellent overlaps.

Summary / Conclusions

• Atomic Bose condensates offer the possibility of studying Bose systems with varying interaction energy (and sign).

• In contrast to superfluid ^{4}He the healing length can be much larger than all other relevant lengthscales.

• Exact diagonalisations indicate that there exist unusual multi-vortex states of rotating Bose condensates, in the limit of large healing length.

• These states can be well-described by a model of non-interacting composite fermions; they are closely related to incompressible fractional quantum Hall liquids.