Effects of Berry Curvature in Ultracold Gases

Nigel Cooper
Cavendish Laboratory, University of Cambridge

"Detecting Topological Order in Cold Atoms"
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Hannah Price & NRC, PRA **85**, 033620 (2012); PRL **111**, 220407 (2013)
Aidelsburger, Lohse, Schweizer, Atala, Barreiro, Nascimbène, NRC, Bloch & Goldman, Nat. Phys. **11**, 162 (2015)

EPSRC
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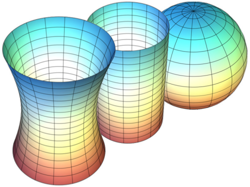
Topological Invariants

Gaussian curvature $\kappa = \frac{1}{R_1 R_2}$



negative, zero and positive κ

$$\frac{1}{2\pi}\int_{\text{closed surface}} \kappa \, dA \quad = \ (2-2g)$$ Gauss-Bonnet Theorem

genus $g = 0, 1, 2, \ldots$ for sphere, torus, 2-hole torus...

Topological invariant: g cannot change under smooth deformations

Topological Features of 2D Bands

[Thouless, Kohmoto, Nightingale & den Nijs (1982)]

Chern number $\mathcal{C} = \frac{1}{2\pi}\int_{\rm BZ} d^2\mathbf{k}\,\Omega_{\mathbf{k}}$

Berry curvature $\Omega_{\mathbf{k}} = -i\nabla_{\mathbf{k}} \times \langle u|\nabla_{\mathbf{k}} u\rangle \cdot \hat{\mathbf{z}}$

Crystal momentum $\mathbf{k}$, Bloch state $|u_{\mathbf{k}}\rangle$

Topological invariant:

$\mathcal{C}$ cannot change under smooth variations of the band

$\mathcal{C}$ can be non-zero in the absence of time-reversal symmetry

Physical Consequences

Topology, $\int \Omega_{\mathbf{k}} d^2 k$

- Integer quantum Hall effect, $\sigma_{xy} = \mathcal{C}\frac{e^2}{h}$ [TKNN (1982)]
- Gapless chiral edge state

e.g. Bragg spectroscopy [Goldman, Beugnon & Gerbier, PRL **108**, 255303 (2012)]

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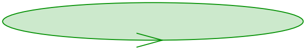


e.g. Bragg spectroscopy [Goldman, Beugnon & Gerbier, PRL **108**, 255303 (2012)]

Geometry, $\Omega_{\mathbf{k}}$

- Expansion imaging [Zhao *et al.*, PRA (2011); Alba *et al.*, PRL (2011); Hauke *et al.*, PRL (2014)]
- Interferometry [Duca *et al.* [LMU], Science **347**, 288 (2015)]

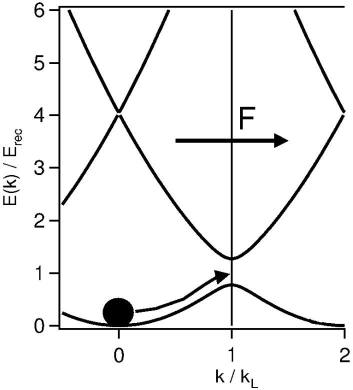
▷ Bloch oscillations [Hannah Price & NRC, PRA **85**, 033620 (2012)]

▷ Collective modes [Hannah Price & NRC, PRL **111**, 220407 (2013)]

Bloch Oscillations (1D)

Wavepacket centered on momentum k and position x

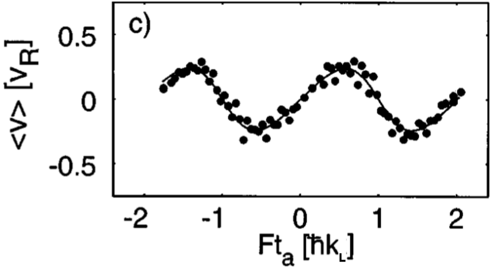
$$\hbar\dot{k} = F$$
$$\dot{x} = \frac{1}{\hbar}\frac{d\varepsilon_k}{dk}$$



Oscillations in $\dot{x}$ (and x) with period $T_{\rm B} = \frac{2\hbar k_L}{F}$

Bloch Oscillations (1D)

Accelerated 1D lattice [Ben Dahan, Peik, Reichel, Castin & Salomon, PRL **76**, 4508 (1996)]



$$\langle \hat{v} \rangle = \frac{1}{m} \langle \hat{p} \rangle$$ from expansion images

Bloch Oscillations in 2D

Modified by the geometry of the Bloch wave functions $|u_{\mathbf{k}}\rangle$

[Chang & Niu, PRL **75**, 1348 (1995)]

$$\begin{aligned} \hbar\dot{\mathbf{k}} &= \mathbf{F} \\ \dot{\mathbf{r}} &= \frac{1}{\hbar}\frac{\partial \varepsilon_{\mathbf{k}}}{\partial \mathbf{k}} - \underbrace{(\dot{\mathbf{k}} \times \hat{\mathbf{z}})\Omega_{\mathbf{k}}}_{\text{anomalous velocity}} \end{aligned}$$

Berry curvature $\qquad \Omega_{\mathbf{k}} = -i\nabla_{\mathbf{k}} \times \langle u|\nabla_{\mathbf{k}} u\rangle \cdot \hat{\mathbf{z}}$

Crystal momentum $\mathbf{k}$, Bloch state $|u_{\mathbf{k}}\rangle$

The physical properties of a band depend on both $\varepsilon_{\mathbf{k}}$ *and* $\Omega_{\mathbf{k}}$

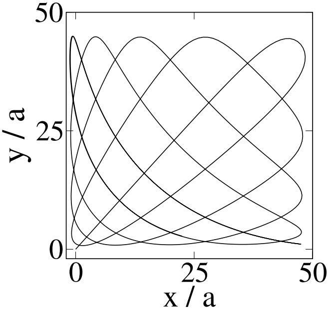
Bloch Oscillations in 2D

$$\hbar\dot{\mathbf{k}} = \mathbf{F} \qquad \dot{\mathbf{r}} = \frac{1}{\hbar}\frac{\partial \varepsilon_{\mathbf{k}}}{\partial \mathbf{k}} - (\dot{\mathbf{k}} \times \hat{\mathbf{z}})\Omega_{\mathbf{k}}$$

Complicated trajectories even for $\Omega_{\mathbf{k}} = 0$

e.g. $\varepsilon_{\mathbf{k}} = -2J\left[\cos k_x a + \cos k_y a\right]$

$$\begin{aligned}\dot{\mathbf{r}} &= \frac{2Ja}{\hbar}(\sin k_x a, \sin k_y a) \\ &= \frac{2Ja}{\hbar}\left(\sin\frac{F_x ta}{\hbar}, \sin\frac{F_y ta}{\hbar}\right)\end{aligned}$$



Lissajous figures when **F** not along a high-symmetry direction

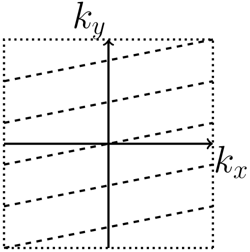
Time-Reversal Protocol

[Hannah Price & NRC, PRA **85**, 033620 (2012)]

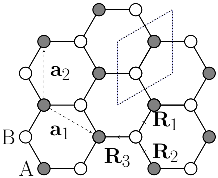
$$\begin{aligned} \hbar\dot{\mathbf{k}} &= \mathbf{F} \\ \dot{\mathbf{r}} &= \frac{1}{\hbar}\frac{\partial \varepsilon_{\mathbf{k}}}{\partial \mathbf{k}} - (\dot{\mathbf{k}} \times \hat{\mathbf{z}})\Omega_{\mathbf{k}} \end{aligned}$$

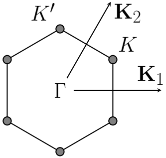
Measure $v_{\mathbf{k}}(+\mathbf{F})$ and $v_{\mathbf{k}}(-\mathbf{F})$

$$\begin{aligned} \mathbf{v}_{\mathbf{k}}(+\mathbf{F}) + \mathbf{v}_{\mathbf{k}}(-\mathbf{F}) &= \frac{2}{\hbar}\frac{\partial \varepsilon_{\mathbf{k}}}{\partial \mathbf{k}} \\ \mathbf{v}_{\mathbf{k}}(+\mathbf{F}) - \mathbf{v}_{\mathbf{k}}(-\mathbf{F}) &= -\frac{2}{\hbar}(\mathbf{F} \times \hat{\mathbf{z}})\Omega_{\mathbf{k}} \end{aligned}$$



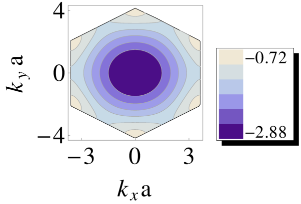
Example: Asymmetric Honeycomb Lattice



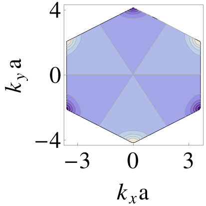


Asymmetric, $V_{\mathrm{A}} = -V_{\mathrm{B}} = W$

Fully gapped



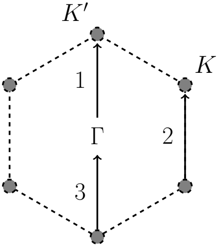
$\Omega_{\mathbf{k}} \neq 0$

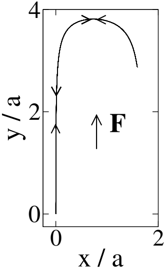


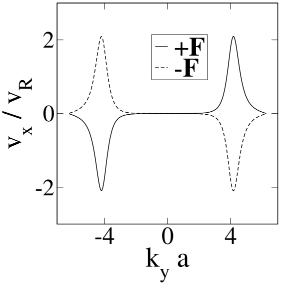
non-zero Berry curvature, but zero Chern number (TRS)

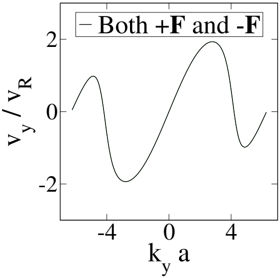
Bloch Oscillations in Asymmetric Honeycomb Lattice

$\mathbf{F} = \left(0, \frac{2}{3}\right) \frac{J}{a}$
$W = \frac{1}{2} J$





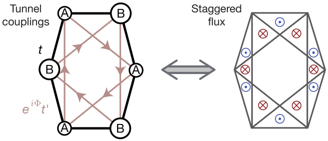




Experimental Results: Chern bands

Haldane Model (fermions)

[Jotzu, Messer, Desbuquois, Lebrat, Uehlinger, Greif & Esslinger, Nature **515**, 237 (2014)]

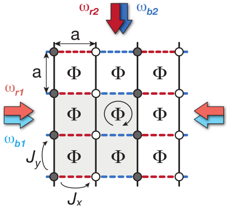


detection of local Berry curvature

[Talk of Rémi Desbuquois]

Harper-Hofstadter model (bosons)

[Aidelsburger, Lohse, Schweizer, Atala, Barreiro, Nascimbène, NRC, Bloch & Goldman, Nat. Phys. **11**, 162 (2015)]

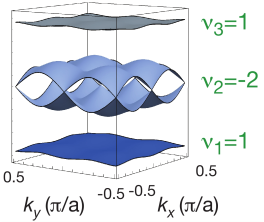


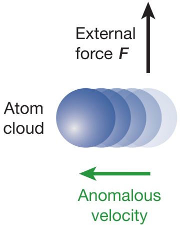
measurement of the Chern number

Measuring the Chern number with Bosons

[Aidelsburger, Lohse, Schweizer, Atala, Barreiro, Nascimbène, NRC, Bloch & Goldman, Nat. Phys. **11**, 162 (2015)]

Flux $n_\phi = 1/4$





$F_y \Rightarrow$ mean transverse velocity, $\bar{v}_x = \frac{1}{N} \int d^2\mathbf{k}\ n_\mathbf{k} \left[\frac{1}{\hbar} \frac{\partial \varepsilon_\mathbf{k}}{\partial k_x} - \frac{F_y}{\hbar} \Omega_\mathbf{k} \right]$

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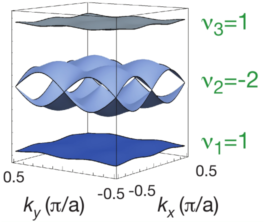

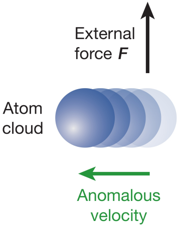

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For $n_\mathbf{k} = \bar{n} \Rightarrow \bar{v}_x = \dfrac{\bar{n}}{N}\displaystyle\int d^2\mathbf{k}\left[\frac{1}{\hbar}\frac{\partial \varepsilon_\mathbf{k}}{\partial k_x} - \frac{F_y}{\hbar}\Omega_\mathbf{k}\right] = 0 - \frac{(2\pi\mathcal{C})}{A_{\mathrm{BZ}}}\frac{F_y}{\hbar}$

integer Chern number, $\mathcal{C} \Rightarrow$ IQHE [Thouless, Kohmoto, Nightingale & den Nijs (1982)]

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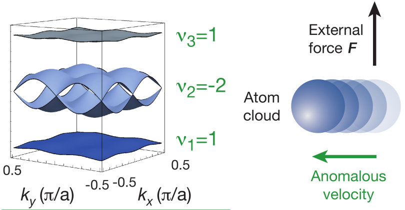
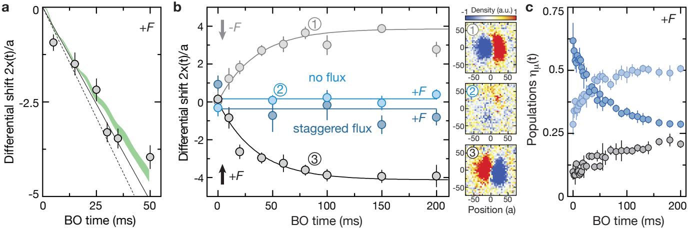
Uniformly populated, $n_{\mathbf{k}} \simeq \bar{n}$





$$\nu_{1,\mathrm{exp}} \simeq 0.99(5)$$

How does Berry curvature affect an *interacting* gas?

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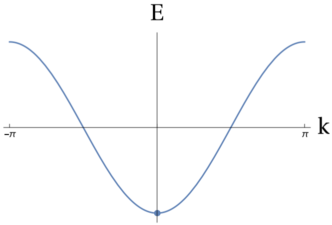
Weakly-interacting BEC

- Condensed in a band minimum
- Effective mass M^* (assume isotropic)
- Berry curvature $\boldsymbol{\Omega} = \Omega \mathbf{e}_z$



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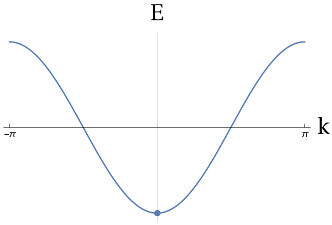

Collective modes are a sensitive probe of the gas [e.g. S. Stringari, PRL (1996)]

Can be used to detect the Berry curvature

[Hannah Price & NRC, PRL **111**, 220407 (2013)]

Hydrodynamic Approach

[e.g. Pethick & Smith]

$$\dot{\rho} + \boldsymbol{\nabla} \cdot (\rho \mathbf{v}) = 0$$

$$\dot{\mathbf{v}} = \frac{\mathbf{F}}{M^*} - \left(\frac{\dot{\mathbf{F}}}{\hbar} \times \hat{\mathbf{z}} \right) \Omega$$

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Thomas-Fermi limit: $\rho_0 = [\mu - V(\mathbf{r})]/g$ (interaction energy ρg)

Linearize, $\rho = \rho_0 + \delta\rho \Rightarrow \mathbf{F} = -g \boldsymbol{\nabla} \delta\rho$

$$\delta\ddot{\rho} = -\frac{\boldsymbol{\nabla} V \cdot \boldsymbol{\nabla} \delta\rho}{M^*} + \frac{\rho_0 g \boldsymbol{\nabla}^2 \delta\rho}{M^*} + \underbrace{\frac{\boldsymbol{\nabla} V \cdot (\boldsymbol{\nabla} \delta\dot{\rho} \times \hat{\mathbf{z}}) \Omega}{\hbar}}_{\text{new term}}$$

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Uniform gas: $\omega = \sqrt{\rho_0 g / M^*} |\mathbf{k}|$, unaffected by Berry curvature.

Harmonically Trapped Gas

$V(\mathbf{r}) = \frac{1}{2}\kappa|\mathbf{r}|^2 \quad \Rightarrow \quad \delta\rho = D(r)Y_{\ell m}(\theta,\varphi)e^{-i\omega t}$

$$\omega = -\frac{m\kappa\Omega}{2\hbar} + \frac{1}{2}\sqrt{\left(\frac{m\kappa\Omega}{\hbar}\right)^2 + \frac{4\kappa}{M^*}(\ell + 3n_r + 2n_r\ell + 2n_r^2)}$$

Berry curvature affects modes with nonzero m

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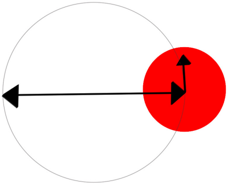
Berry curvature affects modes with nonzero m

e.g. dipole modes $n_r = 0$, $\ell = 1$, $m = \pm 1$ are split by $\Omega \neq 0$

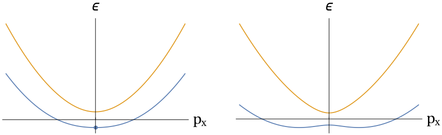
$\Rightarrow$Precession



[cf. Weak magnetic field, LeBlanc *et al.*[NIST], PNAS **109**, 10811 (2012)]

Example: 2D Rashba Spin-Orbit Coupling + Zeeman Field

$$\hat{h}_0 = \frac{\mathbf{p}^2}{2M} + \lambda(p_x\hat{\sigma}_y - p_y\hat{\sigma}_x) - \Delta\hat{\sigma}_z + V(\mathbf{r})$$



$\zeta < 1$ $\zeta > 1$

$$\varepsilon_\pm = \frac{p^2}{2M} \pm \sqrt{\lambda^2|\mathbf{p}|^2 + \Delta^2}$$

$$\zeta \equiv \frac{\lambda^2 M}{\Delta}$$

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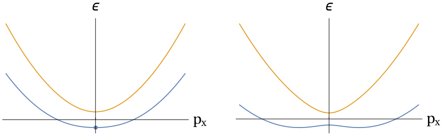

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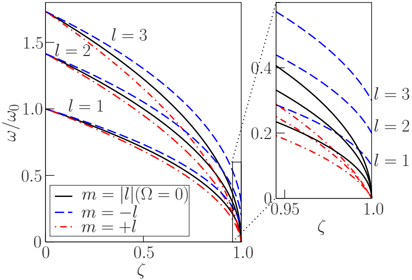
$$\zeta \equiv \frac{\lambda^2 M}{\Delta} < 1$$

$$M^* = \frac{M}{1-\zeta},\ \Omega = \frac{\lambda^2\hbar^2}{2\Delta^2}$$



$\Rightarrow$A sensitive way to measure Ω

Summary

- Two-dimensional bands can have a geometrical character, encoded in a Berry curvature $\Omega_{\mathbf{k}}$.
- The local Berry curvature modifies the Bloch oscillations of an atomic wave packet. This anomalous velocity can be cleanly identified by a "time-reversal" protocol.
- The Berry curvature affects the collective modes of interacting BECs in traps, splitting the frequencies of modes with non-zero angular momentum projection, $m \neq 0$.
- These effects are ubiquitous for cold gases in situations of spin-orbit coupling or for non-primitive optical lattices.