Adiabatic Preparation of Vortex Lattices

Nigel Cooper
Cavendish Laboratory, University of Cambridge

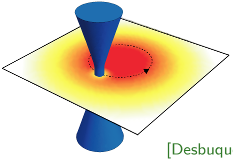
"Synthetic Gauge Fields for Photons and Atoms"
Trento, 1 July 2013

Stefan Baur & NRC, arXiv:1306.4796

EPSRC
Engineering and Physical Sciences
Research Council

Vortex Lattices in Rotating BECs

Stir the atomic gas

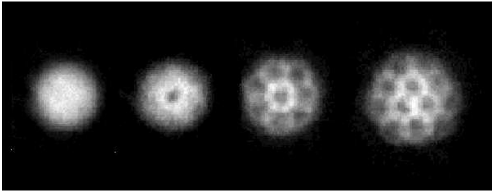


[Desbuquois *et al.* (2012)]

Rotating frame, angular velocity $\boldsymbol{\Omega}$

Coriolis Force $\Leftrightarrow$ Lorentz Force $\Rightarrow$ flux density $n_\phi \equiv \frac{qB}{h} = \frac{2M\Omega}{h}$

$\Rightarrow$ vortex lattice

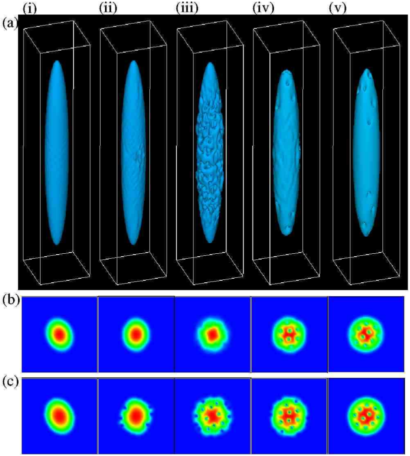


[Madison, Chevy, Bretin & Dalibard, PRL **84**, 806 (2000)]

Vortex Lattice Formation

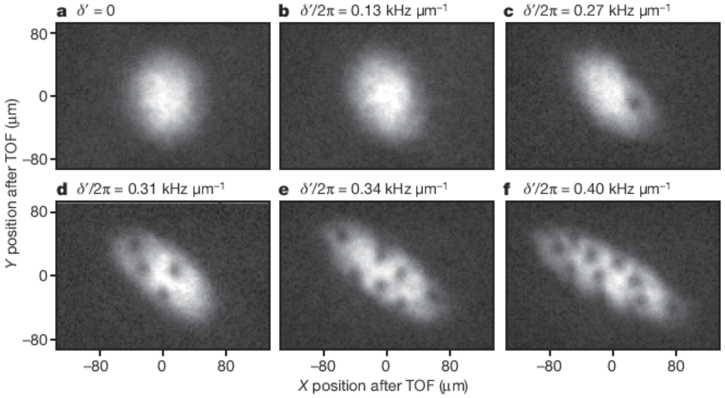
Surface instability [Dalfovo & Stringari, PRA 2000; Madison *et al.*, PRL 2001; Feder *et al.* PRL 2001; Lobo, Sinatra & Castin, PRL 2004]

e.g. [K. Kasamatsu, M. Machida, N. Sasa & M. Tsubota, PRA **71**, 063616 (2005)]



Optically Induced Gauge Fields

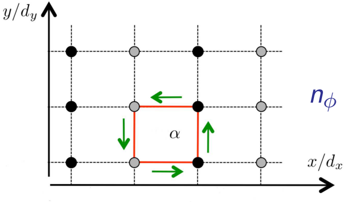
[Y.-J. Lin, R.L. Compton, K. Jiménez-García, J.V. Porto & I.B. Spielman, Nature **462**, 628 (2009)]



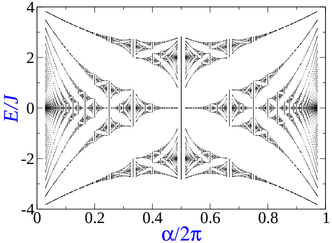
Synthetic Magnetic Fields in Optical Lattices

- Tight-binding lattices + tunneling phases

[Jaksch & Zoller '03; Mueller '04; Sørensen, Demler & Lukin '05; Gerbier & Dalibard '10; Struck *et al.* '12]



$$n_\phi = \frac{\alpha}{2\pi d_x d_y}$$



- "Optical flux lattices"

[NRC '11; NRC & Dalibard '11, '13; Juzeliūnas & Spielman '12]

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \hat{V}(\mathbf{r})$$

- Very high vortex densities $n_\phi \sim \frac{1}{\lambda^2}$

Outline

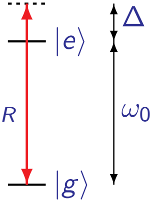
Optically Induced Gauge Fields

[J. Dalibard, F. Gerbier, G. Juzeliūnas, P. Öhberg, RMP **83**, 1523 (2011)]

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \hat{V}(\mathbf{r})$$

Coherent optical coupling of N internal atomic states

laser coupling Ω_R

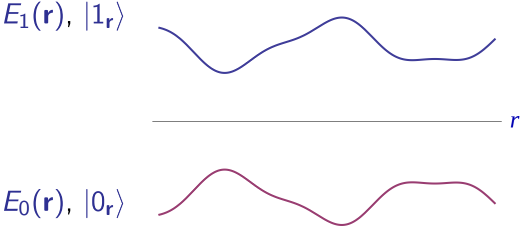


[e.g. $^1S_0$ and $^3P_0$ for Yb or alkaline earth atom]

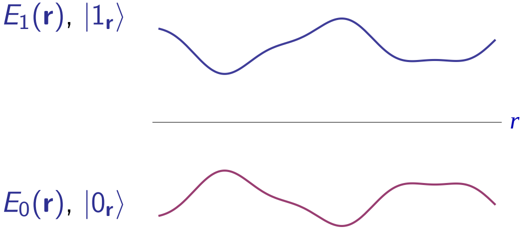
RWA $$\hat{V}(\mathbf{r}) \to \frac{\hbar}{2}\begin{pmatrix} \Delta & \Omega_R(\mathbf{r}) \\ \Omega_R^*(\mathbf{r}) & -\Delta \end{pmatrix}$$

In general $\hat{V}(\mathbf{r}) = \frac{\hbar}{2}\begin{pmatrix} \Delta(\mathbf{r}) & \Omega_R(\mathbf{r}) \\ \Omega_R^*(\mathbf{r}) & -\Delta(\mathbf{r}) \end{pmatrix}$ varying on scale λ

In general $\hat{V}(\mathbf{r}) = \frac{\hbar}{2}\begin{pmatrix} \Delta(\mathbf{r}) & \Omega_R(\mathbf{r}) \\ \Omega_R^*(\mathbf{r}) & -\Delta(\mathbf{r}) \end{pmatrix}$ varying on scale λ



In general $\hat{V}(\mathbf{r}) = \frac{\hbar}{2}\begin{pmatrix} \Delta(\mathbf{r}) & \Omega_R(\mathbf{r}) \\ \Omega_R^*(\mathbf{r}) & -\Delta(\mathbf{r}) \end{pmatrix}$ varying on scale λ

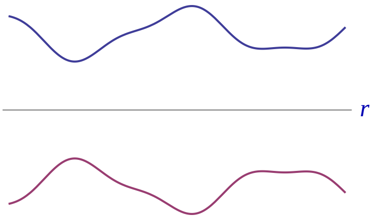


$\text{K.E.} \sim E_\mathrm{R} = \frac{h^2}{2M\lambda^2} \ll E_1 - E_0$: adiabatic motion in state $|0_\mathbf{r}\rangle$

$$|\psi(\mathbf{r})\rangle = \psi_0(\mathbf{r})|0_\mathbf{r}\rangle$$

In general $\hat{V}(\mathbf{r}) = \frac{\hbar}{2}\begin{pmatrix} \Delta(\mathbf{r}) & \Omega_R(\mathbf{r}) \\ \Omega_R^*(\mathbf{r}) & -\Delta(\mathbf{r}) \end{pmatrix}$ varying on scale λ

$E_1(\mathbf{r})$, $|1_\mathbf{r}\rangle$



$E_0(\mathbf{r})$, $|0_\mathbf{r}\rangle$

K.E. $\sim E_\mathrm{R} = \frac{h^2}{2M\lambda^2} \ll E_1 - E_0$: adiabatic motion in state $|0_\mathbf{r}\rangle$

$$|\psi(\mathbf{r})\rangle = \psi_0(\mathbf{r})|0_\mathbf{r}\rangle$$

"Berry connection" $\Rightarrow$vector potential $q\mathbf{A} = i\hbar\langle 0_\mathbf{r}|\boldsymbol{\nabla} 0_\mathbf{r}\rangle$

flux density $n_\phi \equiv \frac{qB}{h} = \frac{1}{h}\boldsymbol{\nabla}\times(q\mathbf{A})$

[J. Dalibard, F. Gerbier, G. Juzeliūnas & P. Öhberg, RMP **83**, 1523 (2011)]

Optical Flux Lattices

Optical lattices of $\hat{V}(\mathbf{r})$ with non-zero mean flux density

$$n_\phi \equiv \frac{qB}{h} = \frac{1}{h}\boldsymbol{\nabla} \times [i\hbar\langle 0_\mathbf{r}|\boldsymbol{\nabla} 0_\mathbf{r}\rangle] \sim \frac{1}{\lambda^2}$$

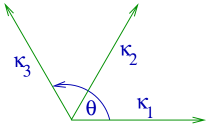
Various implementations:

- 2 electronic states ("clock" transition) [NRC, PRL '11]
- Hyperfine levels (e.g. K, Rb) [NRC & Dalibard, EPL '11; Juzeliūnas & Spielman, NJP '12]
- Beyond SU(2) (3 hyperfine/orbital states) [NRC & Dalibard, PRL '13]

Triangular Optical Flux Lattice

$$\hat{V} = V_0 \left[\hat{\sigma}_x \cos(\kappa_1 \cdot \mathbf{r}) + \hat{\sigma}_y \cos(\kappa_2 \cdot \mathbf{r}) + \hat{\sigma}_z \cos(\kappa_3 \cdot \mathbf{r})\right]$$

[NRC, Phys. Rev. Lett. **106**, 175301 (2011)]

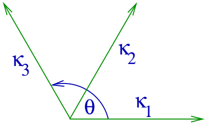


$\theta = 2\pi/3$

Triangular Optical Flux Lattice

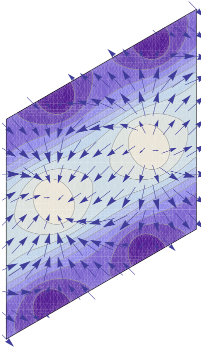
$$\hat{V} = V_0 \left[\hat{\sigma}_x \cos(\kappa_1 \cdot \mathbf{r}) + \hat{\sigma}_y \cos(\kappa_2 \cdot \mathbf{r}) + \hat{\sigma}_z \cos(\kappa_3 \cdot \mathbf{r})\right]$$

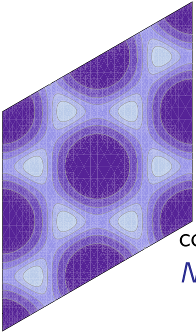
[NRC, Phys. Rev. Lett. **106**, 175301 (2011)]



$\theta = 2\pi/3$

$n_i \equiv \langle 0_{\mathbf{r}} | \hat{\sigma}_i | 0_{\mathbf{r}} \rangle$
vectors: (n_x, n_y)
contours: n_z





contours n_ϕ
$N_\phi = 2$

Expect vortex lattice with $N_\phi = 2$ vortices in this cell

Adiabatic Formation: Essential Idea

[Stefan Baur & NRC, arXiv:1306.4796]

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}}_2 + V_0\left[\hat{\sigma}_x \cos(\kappa_1 \cdot \mathbf{r}) + \hat{\sigma}_y \cos(\kappa_2 \cdot \mathbf{r}) + \hat{\sigma}_z \cos(\kappa_3 \cdot \mathbf{r})\right]$$

Adiabatic Formation: Essential Idea

[Stefan Baur & NRC, arXiv:1306.4796]

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}}_2 + V_0\left[\hat{\sigma}_x \cos(\kappa_1 \cdot \mathbf{r}) + \hat{\sigma}_y \cos(\kappa_2 \cdot \mathbf{r}) + \hat{\sigma}_z \cos(\kappa_3 \cdot \mathbf{r})\right]$$

1. Start with BEC for $V_0 = 0$

Adiabatic Formation: Essential Idea

[Stefan Baur & NRC, arXiv:1306.4796]

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}}_2 + V_0\left[\hat{\sigma}_x \cos(\kappa_1 \cdot \mathbf{r}) + \hat{\sigma}_y \cos(\kappa_2 \cdot \mathbf{r}) + \hat{\sigma}_z \cos(\kappa_3 \cdot \mathbf{r})\right]$$

1. Start with BEC for $V_0 = 0$
2. Ramp up to $V_0 \gtrsim E_{\mathrm{R}}$ $\quad$ $[E_{\mathrm{R}} = \frac{\hbar^2\kappa^2}{2M}]$

Adiabatic Formation: Essential Idea

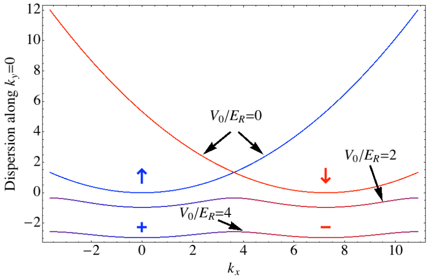
[Stefan Baur & NRC, arXiv:1306.4796]

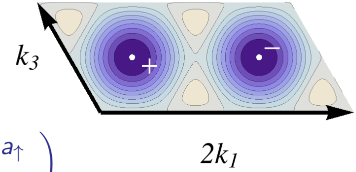
$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}}_2 + V_0\left[\hat{\sigma}_x \cos(\kappa_1 \cdot \mathbf{r}) + \hat{\sigma}_y \cos(\kappa_2 \cdot \mathbf{r}) + \hat{\sigma}_z \cos(\kappa_3 \cdot \mathbf{r})\right]$$

1. Start with BEC for $V_0 = 0$
2. Ramp up to $V_0 \gtrsim E_{\mathrm{R}}$ $\quad [E_{\mathrm{R}} = \frac{\hbar^2\kappa^2}{2M}]$
3. That's it!

More carefully: Bandstructure

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}}_2 + \hat{V}$$





Two degenerate minima
⇒infinite degeneracy

cf. degeneracy of spinor BEC $\sqrt{n_0}\begin{pmatrix} a_\uparrow \\ a_\downarrow \end{pmatrix}$

Break degeneracy

(i) Detuning $\Rightarrow \hat{V}(\mathbf{r}) \to \hat{V}(\mathbf{r}) + \delta\hat{\sigma}_z$

(ii) Weak interactions

$$E_{\rm int} = \int d^2\mathbf{r}\ \frac{g_{\uparrow\uparrow}}{2} n_\uparrow^2(\mathbf{r}) + \frac{g_{\downarrow\downarrow}}{2} n_\downarrow^2(\mathbf{r}) + g_{\uparrow\downarrow} n_\uparrow(\mathbf{r}) n_\downarrow(\mathbf{r})$$

e.g. $g_{\uparrow\downarrow} > g_{\downarrow\downarrow} > g_{\uparrow\uparrow} > 0$ [phase separation, favouring spin ↑]

Initialize the condensate in the spin-↑ state, $\mathbf{k} = 0$

$$\phi = \sqrt{n_0} \left(\begin{array}{c} 1 \\ 0 \end{array} \right)$$

Evolves continuously to eigenstate of "+" minimum

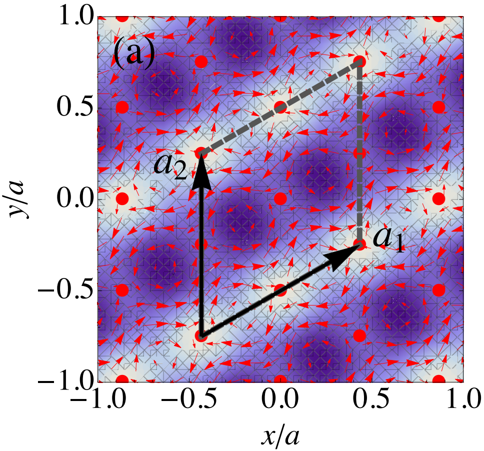
$\Rightarrow$Adiabatic route to a stable vortex lattice

Vortex Lattice?

[Stefan Baur & NRC, arXiv:1306.4796]

particle density (colours)
current density (vectors)

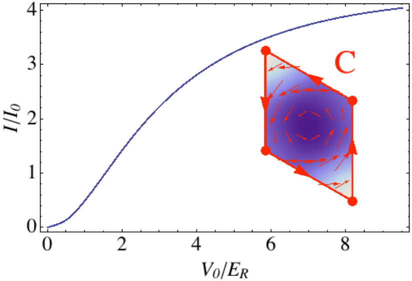
$(V_0 = 4E_{\mathrm{R}})$



Rectangular vortex lattice
(pinned to lattice)

Continuity

Current increases continuously as the lattice depth is increased



+ smooth growth of density modulation

Vortex cores?

Phase singularity

$$\psi(\mathbf{r}) = \psi(r, \theta) \sim r\, e^{i\theta}$$

$\Rightarrow$vanishing density at vortex core

How can a "zero" appear smoothly?

Vortex cores?

Phase singularity

$$\psi(\mathbf{r}) = \psi(r, \theta) \sim r\, e^{i\theta}$$

$\Rightarrow$vanishing density at vortex core

How can a "zero" appear smoothly?

In general $\quad |\psi(\mathbf{r})\rangle = \psi_0(\mathbf{r})|0_\mathbf{r}\rangle + \psi_1(\mathbf{r})|1_\mathbf{r}\rangle$

vortex in component-0 filled by component-1 ("coreless vortex")

[Mermin & Ho, PRL '76]

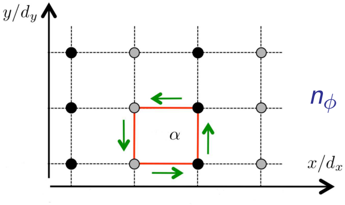
$$V_0 \gg E_\mathbf{R} \qquad |\psi(\mathbf{r})\rangle \to \psi_0(\mathbf{r})|0_\mathbf{r}\rangle$$

Outline

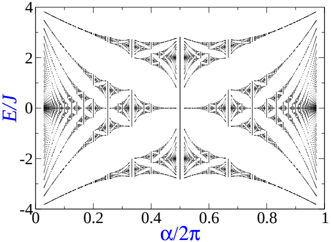
Tight-binding lattice

Imprint phases on tunneling matrix elements

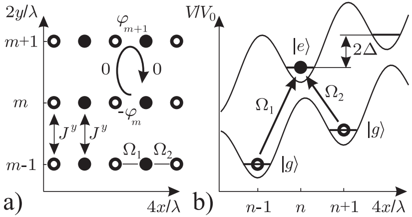
[Jaksch & Zoller '03; Mueller '04; Sørensen, Demler & Lukin '05; Gerbier & Dalibard 2010; Struck *et al.* (2012)]



$$n_\phi = \frac{\alpha}{2\pi d_x d_y}$$

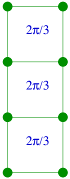


e.g. Jaksch-Zoller scheme



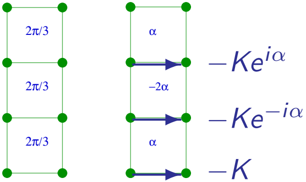
Adiabatic Route: Essential Idea

e.g. $\alpha = 2\pi/3$ magnetic unit cell



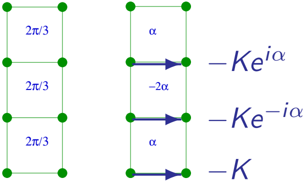
Adiabatic Route: Essential Idea

e.g. $\alpha = 2\pi/3$ magnetic unit cell



Adiabatic Route: Essential Idea

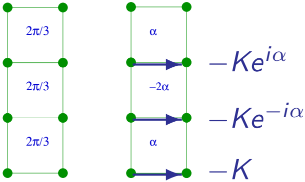
e.g. $\alpha = 2\pi/3$ magnetic unit cell



For <u>fixed unit cell</u>, vary phase $\alpha = 0 \to 2\pi/3$

Adiabatic Route: Essential Idea

e.g. $\alpha = 2\pi/3$ magnetic unit cell

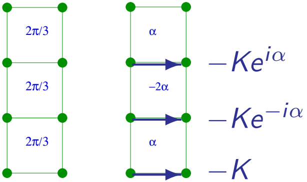


For <u>fixed unit cell</u>, vary phase $\alpha = 0 \to 2\pi/3$

e.g. RF + Raman $Ke^{i\phi} = K_{\rm RF} + K_{\rm Raman} e^{-i\frac{2\pi}{3a}y}$ [I. Bloch]

Adiabatic Route: Essential Idea

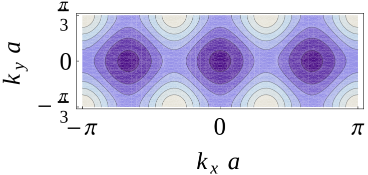
e.g. $\alpha = 2\pi/3$ magnetic unit cell



For <u>fixed unit cell</u>, vary phase $\alpha = 0 \rightarrow 2\pi/3$

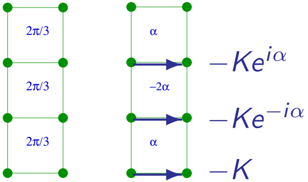
e.g. RF + Raman $Ke^{i\phi} = K_{\mathrm{RF}} + K_{\mathrm{Raman}} e^{-i\frac{2\pi}{3a}y}$ [I. Bloch]

Lowest energy band becomes



Adiabatic Route: Essential Idea

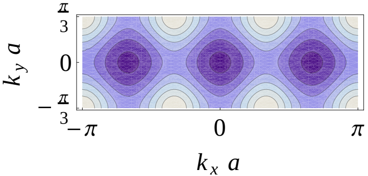
e.g. $\alpha = 2\pi/3$ magnetic unit cell



For fixed unit cell, vary phase $\alpha = 0 \rightarrow 2\pi/3$

e.g. RF + Raman $Ke^{i\phi} = K_{\mathrm{RF}} + K_{\mathrm{Raman}}e^{-i\frac{2\pi}{3a}y}$ [I. Bloch]

Lowest energy band becomes



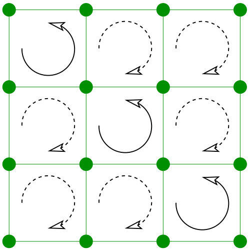
⇒transfer to a BEC in one of these degenerate minima

But... unstable to interactions

Three degenerate minima ⇒BEC in any superposition $\sqrt{n_0}\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$

+ weak repulsive interactions $E_{\rm int} = \frac{1}{2}U\sum_i n_i^2$

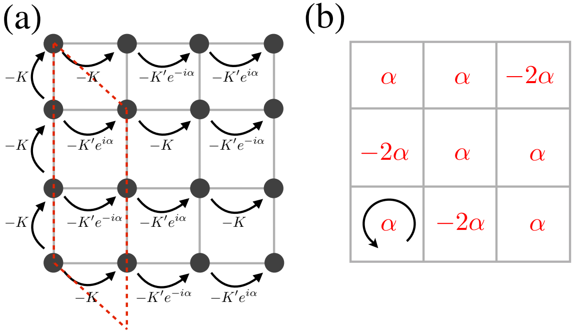
Lowest-energy BEC involves translational symmetry breaking (vortex lattice!)
[Straley & Barnett, PRB '93; Powell *et al.*, PRL '10; Zhang *et al.*, PRL '10]



Naive geometry (1x3) does not load the BEC into this state

Adiabatic route to vortex lattice

(1) Choose unit cell to match target lattice geometry



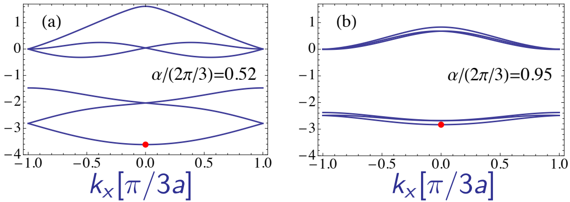
$$K_{x,y} = Ke^{i\theta} r + Ke^{-i(x+y)\frac{2\pi}{3a}}(1-r)$$

(2) Vary $r = 1 \to 0$: uniform BEC $\to$ stable vortex lattice

Evolution

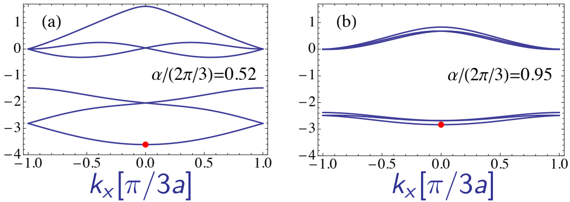
Spectrum

$\epsilon(k_x, k_y = 0)$



Evolution

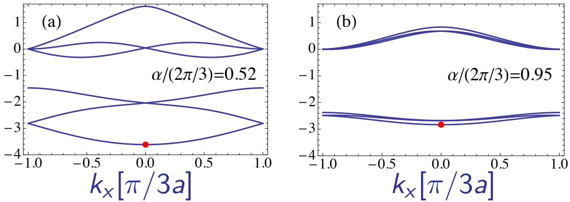
Spectrum
$\epsilon(k_x, k_y = 0)$



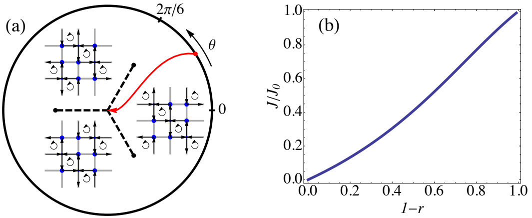
BEC evolves into favoured vortex lattice for $\alpha = \frac{2\pi}{3}$ $(r = 0)$

Evolution

Spectrum
$\epsilon(k_x, k_y = 0)$



BEC evolves into favoured vortex lattice for $\alpha = \frac{2\pi}{3}$ $(r = 0)$



Summary

- Vortex lattices, with $n_\phi \sim 1/\lambda^2$, can be prepared by adiabatic loading of a BEC into lattices with synthetic magnetic fields.
- Vortex lattice current patterns appear smoothly, without the need for vortices to "enter" from the sides.
- Interactions lift degeneracies and select the vortex lattice geometry. A carefully chosen route is needed for adiabaticity.
- A useful starting point for the creation of strongly correlated phases, with $n \sim n_\phi$.