Concepts in Theoretical Physics

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- Aim of course: to develop methods of many-body & (low-energy) quantum field theory motivated by applications!
- Synopsis
- Relation to QFT & other Part III courses?
- Prerequistes? (TPs etc.)
- Lecture notes, problem sets & books

  course web-page:
  `http://www.tcm.phy.cam.ac.uk/~bds10/`
- Supervisions

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How can we develop a Quantum Theory of "Dense" Matter?

the "quantum many-body problem"...

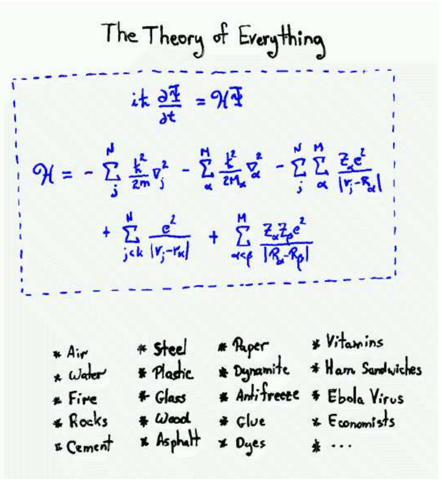
- **First principles?**

according to some,
we already have a **Theory of Everything**...

...but clearly the ability to reduce everything to simple fundamental laws does not imply the ability to start form those laws and reconstruct the universe!

- **Perturbation theory about non-interacting** (i.e. free-particle) **reference state?**

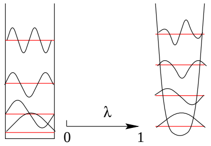
effective, but limited in scope...
consider effect of interactions...



Robert Laughlin (Nobel lecture)

How do particle interactions influence properties of quantum many-particle system?

- **Non-interacting system** ⇝ ideal gas laws
  e.g. free electron theory, Debye theory, etc.
- **Weak interaction**
  ...relies on principle of "adiabatic continuity":

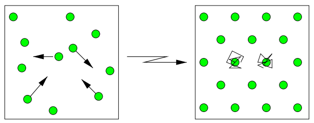


labels of wavefunctions (quantum nos.)[1]
more robust than wavefunctions themselves

⇝ "quasi-particle correspondence"...

[1]...which depend only on fundamental symmetries (translation, rotation, etc.) and dimensionality

How do particle interactions influence properties of quantum many-particle system?



- **Strong interaction** ⇝ symmetry breaking
  (i.e. transitions to new phases of matter)

  e.g. crystal, magnet, superfluid...

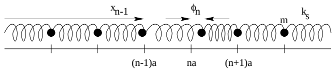
each phase characterised by
new "collective excitations"[2]

e.g. ...

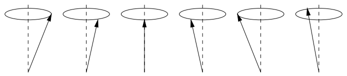
[2]i.e. particle-like excitations involving the collective motion of many elementary particles

Broken symmetry & collective modes

I. **Crystal**: broken translational symmetry
⇝ lattice vibrations (phonons)



II. **Magnet**: broken spin rotational symmetry
⇝ spin waves (magnons)



III. **Superfluid** — broken gauge symmetry
⇝ phase mode

IV. **Quark plasma** — strong interaction
broken gauge symmetry ⇝ hadrons

etc., etc.

...properties of collective excitations usually very different from elementary bare particles

Heirarchical view of matter[1]

- Each phase of matter characterised and classified by fundamental symmetries

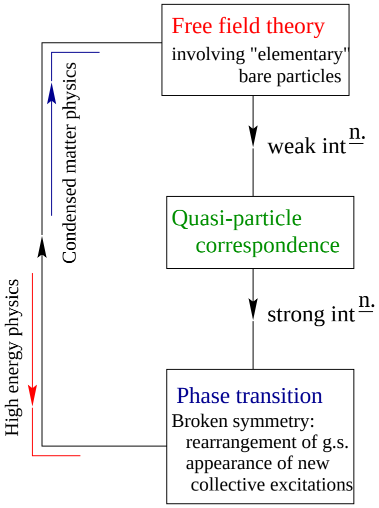
  e.g. translation, rotation, spin, gauge,...
- Within each phase, fluctuations characterised by (particle-like) excitations, quasi-particles, whose properties mirror those of the free theory

  e.g. electron/hole quasi-particles,...
- Transitions between phases signalled by symmetry breaking ⇝ new collective quasi-particle excitations (usually very different from the elementary bare particles)

  e.g. fractional quantum Hall liquid,...

This heirarchy lends itself to the common language of quantum field theory...

[1]...familiar from high energy physics



Quantum Theory of Matter

I. **Microscopic Theory**

(a) Many-particle Hamiltonian...
encodes fundamental particle interactions

(b) ...cast as quantum field theory
via quantum path or field integral

(c) Mean-field theory
broken symmetries $\rightsquigarrow$ global phase structure

(d) Low-energy fluctuations
$\rightsquigarrow$ quantum field theory of collective excitations

II. **Phenomenology**

Fundamental symmetries:
constrain low-energy effective field theory

i.e. Universality!

Aim of Course

...to develop basic theoretical machinery
to address quantum many-body physics:

- Method of second quantisation:
language of many-particle quantum mechanics

- Functional field integral methods
  (a) Feynman path integral
  (b) Coherent state path integral

- [Critical phenomena
& methods of statistical field theory]

Synopsis

- Collective excitations — from particles to fields: Linear harmonic chain & free scalar field theory; classical collective modes, phonons; quantising the classical field; *functional analysis* [3]

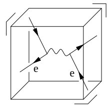
- Method of second quantisation: creation operators; applications to phonons, Mott-Hubbard transition & quantum magnetism; spin wave theory; †antiferromagnetism & weakly interacting Bose gas [5]

- Functional methods: Feynman path integral; *Functional integration & saddle-point analyses*; semi-classics, statistical mechanics; single & double well, instantons & tunnelling; applications to soft matter & quantum friction. [5]

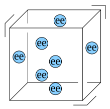
- Functional field theory: *Grassmann Algebra*; coherent state path integral; quantum partition function; Cooper instability & BCS theory; Ginzburg-Landau & statistical field theory; Gauge theory, Higgs mechanism & superconductivity. [5]

- Relativistic quantum mechanics: symmetries & †*Lie groups*; Klein-Gordon & Dirac equation; free particle; Klein paradox; antiparticles; spin; coupling to classical EM field; †particles, interactions & gauge theories. [6]

Superconductivity: A Quantum Phase Transition of the Electron Gas



▷ Exchange of lattice vibrations (phonons) between electrons
$\rightsquigarrow$ (non-local) pairing interaction

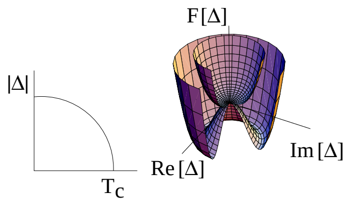


▷ At low T, electrons "condense" in pairs
cf. Bose-Einstein condensation

▷ Phase transition signalled by development of order parameter:

$$\langle \psi_\uparrow \psi_\downarrow \rangle = \Delta \equiv |\Delta| e^{i\phi}$$

possessing both amplitude & phase



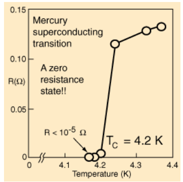
▷ Breaking of (gauge) symmetry...
(cf. ferromagnet, crystal, etc.)

...accompanied by low-energy collective phase fluctuations
(cf. spin waves, phonons, etc.)

▷ Interaction of phase field ϕ with EM-field
$\rightsquigarrow$ photon mass(!) & superconductivity
(cf. Higgs mechanism in electroweak theory)

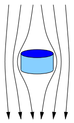
Consequences

▷ Superconductivity:



H. K. Onnes, Commun. Phys. Lab. **12**, 120 (1911)

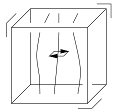
▷ Meissner Effect:



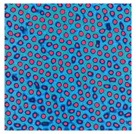
perfect diamagnetic response $\rightsquigarrow$ exclusion of magnetic field

Topological Excitations

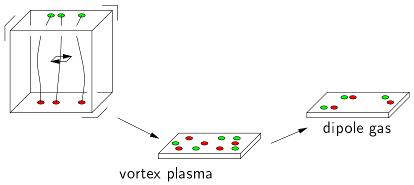
▷ Topological defects: vortices (cf. dislocations)



Vortex lattice



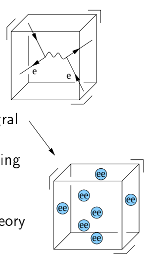
▷ Topological Phase Transition in Thin-films

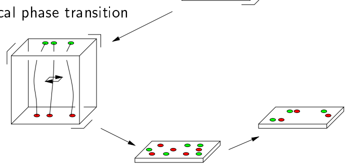


(cf. 2d melting)

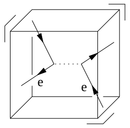
From Microscopics to Quantum Fields

1. Many-body Hamiltonian: "second quantization"
2. Quantum field integral: coherent state path integral
3. Mean-field: symmetry breaking & BCS transition
4. Fluctuations: gauge field theory
5. Topological phase transition





Coulomb Interaction



▷ Electronic Phases of Matter:

Landau Fermi-liquid
Wigner crystal
Mott Insulator
Quantum magnetism & spin liquids
Quantum Hall fluids
Heavy fermion superconductor
Luttinger liquid
Exciton insulator & condensate

Books

▷ **Quantum Many-body theory**

*R. P. Feynman, *Statistical Mechanics*, Benjamin, New York, (1972).

N. Nagaosa, *Quantum Field Theory in Condensed Matter Physics*, Sringer 1999.

*J. W. Negele and H. Orland, *Quantum Many-Particle Systems*, Addison-Wesley Publishing, 1988.

▷ **Functional Field Integral methods**

*A. Das, *Field Theory: A Path Integral Approach*, World Scientific Publishing, (1993).

R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals*, McGraw-Hill, New York, (1965).

*L. S. Schulman, *Techniques and Applications of Path Integration*, John Wiley & Sons, 1981.

▷ **Relativistic Quantum Mechanics**

J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics*, McGraw-Hill (1964).

*L. H. Ryder, *Quantum Field Theory*, Cambridge University Press, 1996.