Summary of Lecture 11

- Boson/Fermion Field Operators (position basis)

$$[\hat{\psi}(\vec{r}), \hat{\psi}^\dagger(\vec{r}')]_\mp = \delta(\vec{r} - \vec{r}') \qquad [\hat{\psi}(\vec{r}), \hat{\psi}(\vec{r}')]_\mp = [\hat{\psi}^\dagger(\vec{r}), \hat{\psi}^\dagger(\vec{r}')]_\mp = 0$$

- Density operator $\hat{\rho}(\vec{r}) = \hat{\psi}^\dagger(\vec{r})\hat{\psi}(\vec{r})$
- Single-particle density matrix $g(\vec{r}, \vec{r}') = \langle \hat{\psi}^\dagger(\vec{r})\hat{\psi}(\vec{r}') \rangle$
- Density-density correlations

$$\langle \hat{\rho}(\vec{r})\hat{\rho}(\vec{r}') \rangle = \langle \hat{\psi}^\dagger(\vec{r})\hat{\psi}^\dagger(\vec{r}')\hat{\psi}(\vec{r}')\hat{\psi}(\vec{r}) \rangle + \delta(\vec{r} - \vec{r}')\langle \hat{\psi}^\dagger(\vec{r})\hat{\psi}(\vec{r}) \rangle$$

$$= \langle : \hat{\rho}(\vec{r})\hat{\rho}(\vec{r}') : \rangle + \delta(\vec{r} - \vec{r}')\langle \rho(\vec{r}) \rangle$$

This Lecture (12)

- Two-body operators and Interactions
- Bose-Hubbard Model
- Bogoliubov Transformation

Summary of Lecture 12

- Two-body operators

$$\frac{1}{2}\sum_{i\neq j} U(\vec{r}_i, \vec{r}_j) \longrightarrow \frac{1}{2}\int \mathrm{d}\vec{r}\mathrm{d}\vec{r}'\ \hat{\psi}^\dagger(\vec{r})\hat{\psi}^\dagger(\vec{r}')\hat{\psi}(\vec{r}')\hat{\psi}(\vec{r})\ U(\vec{r}, \vec{r}')$$

- Bose Hubbard model $\hat{H} = -J\sum_{\langle i,j\rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1)$
- Bose-Einstein condensate $|\mathrm{BEC}\rangle = \frac{1}{\sqrt{N!}}\left(\hat{\tilde{a}}_0^\dagger\right)^N |\mathrm{vac}\rangle$
- Mott Insulator $|\mathrm{Mott}\rangle = \prod_i \hat{a}_i^\dagger |\mathrm{vac}\rangle$
- Bogoliubov Theory $\hat{H}_\mathrm{B} = \sum_k \tilde{\epsilon}_k \hat{\tilde{a}}_k^\dagger \hat{\tilde{a}}_k + \frac{Un}{2}\sum_k \left(\hat{\tilde{a}}_k^\dagger \hat{\tilde{a}}_{-k}^\dagger + \hat{\tilde{a}}_{-k}\hat{\tilde{a}}_k\right)$
- Time of flight, Interference & the single-particle density matrix

Next Lecture (13)

- Density Operators