PROLOGUE
Bouncing Ball

Before tackling a problem every theoretical physicist tries to draw a sketch of its solution. At this stage numerical factors are not important and, therefore, a fine technique is not needed too. The really important thing is to find the major factors and estimate their effects. An example of this sort of analysis forms the prologue to the Fairy Tales. We will discuss an elastic ball, bouncing on a horizontal plane, elastic deformation in the area of their contact, duration of impact, adhesion to the plane and the energy losses in the impact process. Finally, we estimate how long the ball bounces.

Hertz contact problem

Consider a ball of a radius R, pressed by a force F to a plane. Let the ball and the plane have the same Young modules E. As a result of deformation, there will appear a spot of radius a on the plane and a dent of comparable

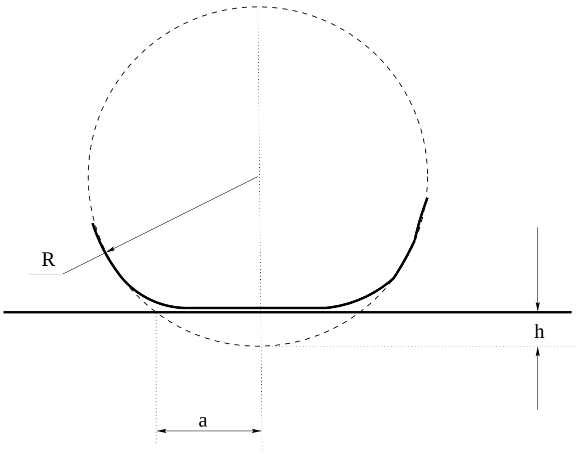

Figure 1: Hertz contact problem

radius on the ball. The compression h of the ball, its radius R and the radius a of the spot are connected as

$$a \sim \sqrt{R^2 - (R-h)^2} \sim \sqrt{hR}.$$

Now we would like to express the compression h through the force F. Using Hooke's law, we equate the strain of shear deformation $\epsilon \sim h/a \sim \sqrt{h/R}$ to the stress $\sigma \sim F/a^2 \sim F/Rh$, divided by the Young modulus E:

$$\epsilon = \frac{\sigma}{E}.$$

As a result

$$h \sim R\left(\frac{F}{ER^2}\right)^{2/3}, \quad F \sim Eh^{3/2}R^{1/2}. \tag{1}$$

Impact

If a ball hits the plane with a speed v, its kinetic energy $mv^2/2$ is transformed into the potential energy of deformation

$$W_P \sim Fh \sim Eh^{5/2}R^{1/2}$$

Since the mass of the ball is $m \sim \rho R^3$, where ρ is its density, the energy conservation law gives an estimate for the maximum compression h_{max} in the collision process

$$h_{\text{max}} \sim R\left(\frac{v}{s}\right)^{4/5},$$

where $s = e/\rho$ is the speed of sound. The time τ_{imp} of the impact is

$$\tau_{\text{imp}} \sim \frac{h_{\text{max}}}{v} \sim \frac{R}{v}\left(\frac{v}{s}\right)^{4/5}$$

This result look plausible, because if $v \sim s$, then $h_{\text{max}} \sim R$. Note that the impact time varies with speed very slowly:

$$\tau_{\text{imp}} \sim v^{-1/5}$$

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Sticking to plane

If the ball sticks to the plane, then the surface energy W_A of adhesion is

$$W_A \sim -\alpha\ a^2,$$

where α is the surface energy per unit area and a^2 is the area of contact. Therefore the total energy is the sum of this surface term W_A and the energy of elastic deformation:

$$W \sim -\alpha\ a^2 + E\ h^{5/2}\ R^{1/2} \tag{2}$$

At the minimum of W $a^2 \sim Rh$ and both terms in Eq (2) are of the same order:

$$\alpha\ a^2 \sim \alpha\ h\ R \sim E\ h^{5/2}\ R^{1/2}$$

Thus, when the ball sticks to the plane, its compression h_A is

$$h_A \sim R \left(\frac{\alpha}{E\ R} \right)^{2/3}$$

and the area of contact is

$$A \sim R^2 \left(\frac{\alpha}{E\ R} \right)^{2/3} .$$

The last two expressions contain the ratio of the Laplace pressure α/R to the Young module E. Thus, at equilibrium, the energy of adhesion is

$$W_A^{(0)} \sim \alpha\ R^2 \left(\frac{\alpha}{E\ R} \right)^{2/3}$$